## **Tailored Stories**

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#### Narratives Matter

Different narratives lead to different conclusions

Voters may disagree on the outcome of an election

"The election system is fair" vs. "Elections are rigged"

Consumers often differ in evaluating a company

"This company has great CSR" vs. "The company is just green washing"

Investors make different predictions based on past financial data

"What goes down comes up" vs. "Early success predicts long-run success"

## This Paper

To which extent is possible to manipulate beliefs with narratives (models)?

1. Without controlling the information to be observed

unlike cheap talk and Bayesian persuasion

- 2. Without knowing the information to be interpreted unlike Schwartzstein & Sunderam (2021)
- Main result characterizes the extent of belief manipulability
  - The agent might hold inconsistent beliefs across contingencies
  - Extent to which the agent is vulnerable to persuasion depends on initial beliefs
- Belief polarization is inevitable with conflicting narratives

### Agenda

- 1. Set-Up
- 2. What can the agent be persuaded of?
- 3. Which phenomena can be explained?

## Set-Up

#### Example













#### Model



Model 
$$\begin{cases} \Pr(W|B) \\ \Pr(W|\neg B) \end{cases}$$

- $B: \quad {\rm Bob} \ {\rm is \ the \ legitimate \ president}$
- W: Bob is the reported winner of the election



#### Model Adoption

Fit of a model *m* conditional on the signal *s*: how likely a model fits the data

$$\operatorname{Pr}^{m}(s) = \operatorname{Pr}(B)\operatorname{Pr}^{m}(s|B) + \operatorname{Pr}(\neg B)\operatorname{Pr}^{m}(s|\neg B)$$

1. Receiver selects the model with the highest fit given the observed signal among the proposed models M by the sender

$$m^*_s \in rgmax_{m \in M} \operatorname{Pr}^m(s)$$

2. Then, she updates her prior via Bayes rule and she chooses the action that maximizes her expected utility calculated using that model







What can the agent be persuaded of?

#### One Model



B: Bob is the legitimate president

W: Bob is the reported winner of the election

F: Fair elections

#### One Model

BOB WON!

BOBIOST





B: Bob is the legitimate presidentW: Bob is the reported winner of the election

F: Fair elections

#### One Model

BOB WON!



 $\Pr^{\mathrm{F}}\left(B|W\right) \bigstar$ 

B: Bob is the legitimate president

W: Bob is the reported winner of the election





 $\Pr^{\mathrm{F}}\left(B|\neg W\right) \bigvee$ 

#### F: Fair elections

#### Vector of Posterior Beliefs

Vector of posterior beliefs: array of posterior distributions conditional on each signal

- Axis: posterior of B conditional on each signal
  - x-axis: posterior probability of B given W
  - y-axis: posterior probability of B given  $\neg W$
- Every model induce a vector of posteriors
- Orange point: prior on B = 0.7
- Red point: vectors of posteriors by fair model



#### Feasible Vectors of Posteriors: One Model

A vector of posterior beliefs is **Bayes-consistent** if the prior is a convex combination of the posterior across signals

- Equivalent representation between models and Bayes-consistent vectors of posteriors
  - Every point in the purple area corresponds to a model
- With a model, the sender can only induce Bayes-consistent vectors of posteriors
  - Comparable characterizing condition to Bayes-plausibility (Kamenica & Gentzkow, 2011)





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# BOB WON!

 $\Pr(B) = 70\%$ 







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 $\mathrm{Pr}^{\mathrm{F}}\left(\mathcal{W}
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$$\begin{split} \Pr^{\mathrm{F}}\left(\mathcal{W}\right) &= 70\% > 16\% = \Pr^{\mathrm{C}}\left(\mathcal{W}\right) \\ &\Rightarrow \Pr^{\mathrm{F}}\left(\mathcal{B}|\mathcal{W}\right) \approx 100\% \uparrow \end{split}$$

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 $\mathrm{Pr}^{\mathrm{F}}\left( \neg \mathcal{W} 
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B: Bob is the legitimate presidentW: Bob is the reported winner of the election



$$\begin{split} \Pr^{\mathrm{F}}\left(\neg W\right) &= 30\% < 84\% = \Pr^{\mathrm{C}}\left(\neg W\right) \\ &\Rightarrow \Pr^{\mathrm{C}}\left(B|\neg W\right) = 82\% \bigstar$$

#### Model Adoption

With more models, the receiver selects the model with the highest fit given the signal

- Red point: vectors of posteriors by fair model
- Blue point: vectors of posteriors by conspiracy theory



With more models, the receiver selects the model with the highest fit given the signal

- Isofit line: all the points correspond to models that have the same fit (except the prior)
  - The steeper, the higher fit given W
  - The flatter, the higher fit given  $\neg W$
- Red line: vectors of posteriors by all models that explain the signals as well as the fair model



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Which model is adopted given W?
 The model on the steepest isofit line
 → Fair model (red)!



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- Which model is adopted given W?
   The model on the steepest isofit line
   → Fair model (red)!
- Which model is adopted given ¬W?
   The model on the flattest isofit line
   → Conspiracy theory (blue)!



Bayesian More Models

### Feasible Vectors of Posteriors: Many Models

Full characterization of the set of feasible vectors of posteriors: on average, the posteriors across signals should not be too far away from the prior

- Bayes-inconsistent vectors of posteriors could be induced
- Not all vectors of posteriors are feasible



#### Sketch of the Proof

A vector of posteriors is feasible if and only if there exists a set of tailored models

- A model is tailored to a specific signal realization if
  - 1. Each model is tailored to a specific signal realization
    - $\rightarrow$  target posterior conditional on that signal
  - 2. Each model is be adopted conditional on the signal it has been tailored to  $\rightarrow$  compatibility constraint for models
    - The higher fit a model has while inducing the target posterior, the more freedom to induce posteriors conditional on other signals using other models
    - The frontier of the feasibility set is generated by maximal overfitting

Such set of models exists if theorem condition is satisfied, otherwise it does not

#### **Comparative Statics**

Initial beliefs drive the extent to which the receiver is vulnerable to persuasion

- The closer the prior is to 50-50, the more belief manipulability
- Full manipulability at 50-50



Which phenomena can be explained?

#### Applications

- Firehose of Falsehood: model of Russian propaganda based on a large number of contradictory and mutually inconsistent messages (Paul & Matthew, 2016)
  - Effective disinformation campaign leading to polarization of heterogeneous voters
- Financial Advice
- Lobbying
- Self-Persuasion

#### Firehose of Falsehood

Voters are exposed to multiple models before elections

- Red point: vectors of posteriors by fair model
- Blue point: vectors of posteriors by conspiracy theory



## Firehose of Falsehood

#### Inevitable Polarization



- With conflicting models, belief polarization occurs
  - There is a threshold in prior such that voters with prior higher (lower) than the threshold would hold extreme high (low) posteriors regardless of the election outcome



## Firehose of Falsehood: 2020 US Presidential Elections



RIGGED 2020 ELECTION: MILLIONS OF MAIL-IN BALLOTS WILL BE PRINTED BY FOREIGN COUNTRIES, AND OTHERS. IT WILL BE THE SCANDAL OF OUR TIMES!

7:16 AM · Jun 22, 2020



The United States cannot have all Mail In Ballots. It will be the greatest Rigged Election in history. People grab them from mailboxes, print thousands of forgeries and "force" people to sign. Also, forge names. Some absentee OK, when necessary. Trying to use Covid for this Scam!

7:08 AM May 24, 2020



With Universal Mail-In Voting (not Absentee Voting, which is good), 2020 will be the most INACCURATE & FRAUDLENT Election in history. It will be a great embarrassment to the USA. Delay the Election until people can properly, securely and safely vote???





NORTH CAROLINA: To make sure your Ballot COUNTS, sign & send it in EARLY. When Polls open, go to your Polling Place to see if it was COUNTED. IF NOT, VOTE! Your signed Ballot will not count because your vote has been posted. Don't let them illegally take your vote away from you!

9:10 AM · Sep 12, 2020

#### Firehose of Falsehood: 2020 US Presidential Elections

Predictions: when exposed to conflicting models...

- 1. Voters with different priors adopt different models once the outcome is observed
- 2. Voters with similar priors adopt different models once observed different outcomes

Stylized facts: Persily and Stewart (2021)

Assumption: voters expect their partisan candidate to win

- Republicans expect Trump to win
- Democrats expect Biden to win



## Stylized Fact 1: Accuracy of Vote Count Persily and Stewart (2021)



More

## Stylized Fact 2: Confidence in Vote Count by State, Republicans Persily and Stewart (2021)



### Other Applications

#### Firehose of Falsehood

- Financial Advice: conflict of interest in finance between a financial advisor and an investor with private information (past experience)
  - The advisor can manipulate the investor regardless of her private signal, always moving her beliefs in the advantageous direction
- Lobbying: challenge a well-established way of looking at scientific evidence, so-called "Merchants of Doubt" (e.g., Michaels, 2008; Oreskes & Conway, 2011)
  - Given a shared default model (trust in science), it is still possible to insinuate doubt
- Self-Persuasion: an agent can distort her own beliefs by manipulating the perceived informativeness of observable signals
  - Leaving facts open to interpretation can be used to achieve self-serving beliefs
  - Distorting confidence using models can motivate in committing to a costly action



- It is possible to persuade others by providing interpretations of unknown events, either future or private information
- It can lead the receiver to hold inconsistent beliefs across signal realizations
  - Each signal might trigger the adoption of a tailored model to manipulate beliefs conditional on that signal
- This form of persuasion sheds light on a mechanism common to inter-personal (conflict of interest in finance, polarization, and lobbying) or intra-personal phenomena (self-persuasion)

Thank you! chiaraaina@fas.harvard.edu

## Appendix

### Literature Review

#### Literature on Narratives

- Models: Schwartzstein & Sunderam (2021, 2023), Ichihashi & Meng (2021), Ispano (2022), Yang (2022), Barron & Fries (2023)
- Directed Acyclical Graphs: Eliaz and Spiegler (2020), Eliaz, Spiegler, & Weiss (2021), Eliaz, Galperti, & Spiegler (2022), Horz & Kocak (2022)
- Others: Benabou, Falk, & Tirole (2018), Levy & Razin (2020), Izzo, Martin, & Callander (2021), Olea, Montiel, Ortoleva, Pai, & Prat (2022), Graeber et al. (2022), Bursztyn et al. (2022), Andre et al. (2022, 2021), Morag and Loewenstein, (2021), ...

#### Literature on Persuasion

- Cheap Talk: Milgrom (1981), Crawford & Sobel (1982), Eliaz, Spiegler, & Thysen (2019), Gleyze & Pernoud (2022), Kellner & Le Quement, (2018, 2017), ...
- Bayesian Persuasion: Kamenica & Gentzkow (2011), Alonso & Camara (2016), Ely (2017), Galperti (2019), Ball & Espín-Sánchez (2022), Beauchêne, Li, & Li (2019), ...



#### The sender communicates models without knowing the signal realization

- 1. Temporal interpretation: the signal realizes after the sender communicates
  - A voter will observe the outcome of the election and a politician wants to be recognized as legitimate president regardless

#### 2. Private information: the signal is only private information of the receiver

• An investor had either good or bad experience on the financial market and a financial advisor wants to convince her to invest in an asset regardless

lsofit: the set of vectors of posteriors induced by models with the same fit levels conditional on every signal realization

Consider the Bayes-consistency constraint for *B* for model *m*:

 $\Pr(B) = \Pr^{m}(W) \,\Pr^{m}(B|W) + \Pr^{m}(\neg W) \,\Pr^{m}(B|\neg W)$ 

Re-arrange:

$$\Pr^{m}(B|\neg W) = \frac{\Pr(B)}{\Pr^{m}(\neg W)} - \frac{\Pr^{m}(W)}{\Pr^{m}(\neg W)} \Pr^{m}(B|W)$$

Slope  $-\frac{\Pr^m(W)}{1-\Pr^m(W)}$ : the higher  $\Pr^m(W)$ , the steeper the line

The receiver adopts the model that fits best the observed signal among the models provided by the sender and uses only that to update her beliefs

- Inference to the Best Explanation (Harman, 1965) only the best hypothesis is used to make inference
  - Hypotheses are supported by the same observations they are supposed to explain (Lipton, 2003; Keil, 2006; Douven et al., 2015)
  - The better an hypothesis explains the data, the more confidence in it (Koehler, 1991; Pennington and Hastie, 1992; Lombrozo and Carey, 2006)
- Under ambiguity aversion, learning efficiency is maximal with Maximum Likelihood updating (Frick et al., 2022)

#### **Bayesian Beliefs**

Bayesian Posterior: average of the posteriors given each model weighted by the probability of each model given the observed signal (based on priors over models)

▶ Priors over models:  $\rho \in \Delta(M)$  with  $\rho^m$  is the prior over model *m* 

Probability of each model *m* once signal *s* is observed:

$$\rho_s^m = \frac{\rho^m \operatorname{Pr}^m(s)}{\sum_{m' \in M} \rho^{m'} \operatorname{Pr}^{m'}(s)}$$

Bayesian posterior of state ω given signal s is calculated as

$$\Pr(\omega|s) = \sum_{m \in M} \rho_s^m \Pr^m(\omega|s))$$

What is the resulting vector of posteriors if the receiver is exposed to many models?

Enough to anticipate the model adopted conditional on each signal

- Which model is adopted given W? The model on the steepest isofit line  $\rightarrow$  Fair model (red)!
- Which model is adopted given  $\neg W$ ? The model on the flattest isofit line  $\rightarrow$  Conspiracy theory (blue)!
- What if more than two models?



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- What if Bayesian?













#### Set of Feasible Vectors of Posteriors: Many Models

► States: 
$$\omega \in \Omega$$
 w/ prior  $\mu_0 \in int(\Delta(\Omega))$ 

Signals:  $s \in S$ 

Vector of posterior beliefs: μ = (μ<sub>s</sub>)<sub>s∈S</sub> ∈ [Δ(Ω)]<sup>S</sup> array of posterior distributions conditional on each signal realization

• Maximal movement for 
$$\mu_s$$
:  $\overline{\delta}(\mu_s) = \max_{\omega \in \Omega} \frac{\mu_s(\omega)}{\mu_0(\omega)}$ 

measure of how much the target posterior is far from the prior in a state

#### Theorem

A vector of posteriors  $\mu$  is feasible if and only if  $HM(\bar{\delta}(\mu_s)) \leq |S|$ the harmonic mean of the maximal movement across signals is not higher than # of the signals

#### Fit vs Movement



#### Lemma

Fix a posterior  $\mu_s$ 

- ▶ For every  $p \in [0, \overline{\delta}(\mu_s)^{-1}]$ , there exists a model inducing  $\mu_s$  with  $\Pr^m(s) = p$
- ▶ Every model inducing  $\mu_s$  has  $\Pr^m(s) \in \left[0, \overline{\delta}(\mu_s)^{-1}\right]$
- Schwartzstein & Sunderam (2021) characterize the upper bound: the maximal fit coincides with the reciprocal of the maximal movement

#### Lemma: Proof

(i) For every  $p \in [0, \overline{\delta}(\mu_s)^{-1}]$ , there exists a model inducing  $\mu_s$  with  $\Pr^m(s) = p$ 

- ▶ Show that for every  $p \in [0, \bar{\delta}_s(\mu_s)^{-1}]$ , there exists a model inducing  $\mu_s$  with  $\Pr^m(s) = p$
- Construct  $\mu$  such that (i)  $\mu_s$  is induced conditional on s, and (ii) for each state  $\omega$ , there exists  $\sigma(s') \in \Delta(S)$  with the additional property  $\sigma(s) = p$  such that Bayes-consistency holds:

$$\sum_{s'} \mu_{s'}(\omega) \ \sigma(s') = \mu_s(\omega) \ \sigma(s) + \sum_{s' \neq s} \mu_{s'}(\omega) \ \sigma(s') = \mu_0(\omega).$$
(a)

- By Lemma 1, there exists a model that induce this Bayes-consistent vector of posteriors with fit p
- There exists multiple vectors of posteriors that satisfy condition (a) as long as, for each  $\omega$ ,

$$\mu_0(\omega) - \mu_s(\omega) \ p = \sum_{s' \neq s} \mu_{s'}(\omega) \ \sigma(s') \ge 0.$$
 (b)

### Lemma: Proof



(i) For every  $p \in [0, \overline{\delta}(\mu_s)^{-1}]$ , there exists a model inducing  $\mu_s$  with  $\Pr^m(s) = p$ 

► For instance, fix a signal  $s'' \neq s$  and, for each  $\omega$ , let  $\mu_{s''}(\omega) = \frac{\mu_0(\omega) - p \ \mu_s(\omega)}{1-p}$ 

Condition (a) is satisfied for the distribution  $\sigma(s')$  such that  $\sigma(s) = p$ ,  $\sigma(s'') = 1 - p$ , and  $\sigma(s') = 0$  for all the other signals

• Condition (b) is implied by  $p \in [0, \overline{\delta}_s(\mu_s)^{-1}]$ a: holding for every state, then

$$p \leq rac{\mu_0(\omega)}{\mu_s(\omega)} \leq \left(rac{1}{\displaystyle\max_{\omega} rac{\mu_s(\omega)}{\mu_0(\omega)}}
ight)^{-1} = ar{\delta}_s(\mu_s)^{-1}$$

#### Lemma: Proof

#### (ii) Every model inducing $\mu_s$ has $\Pr^m(s) \in \left[0, \bar{\delta}(\mu_s)^{-1}\right]$

- Consider an arbitrary model inducing  $\mu_s$  conditional on s
- It follows from Bayes rule that the fit of any *m* inducing the target μ<sup>m</sup><sub>s</sub> = μ<sub>s</sub> conditional on s must be such that, for every ω

$$\mathrm{Pr}^{m}(s) = rac{\mu_{0}(\omega)}{\mu_{s}(\omega)}\pi^{m}(s|\omega)$$

▶ Notice that if  $\pi^m(s|\omega) = 0$  the fit equals 0 (minimal fit). Instead, if  $\pi^m(s|\omega) = 1$ , it follows that

$$\mathrm{Pr}^m(s) \leq rac{\mu_0(\omega)}{\mu_s(\omega)}$$

Because this holds for every state, the maximal fit for μ<sub>s</sub> is the minimum of the ratio across states, which equals the reciprocal of the maximal movement for μ<sub>s</sub>:

$$\min_{\omega} \frac{\mu_0(\omega)}{\mu_s(\omega)} = \frac{1}{\max_{\omega} \frac{\mu_s(\omega)}{\mu_0(\omega)}} = \bar{\delta}_s(\mu_s)^{-1}$$

• The fit of a model that induces the target posterior can only take values in  $[0, \bar{\delta}_s(\mu_s)^{-1}]$ 

### **Comparative Statics**



#### Proposition

## If $\min_{\omega\in\Omega}\mu_0(\omega)\geq rac{1}{|S|}$ , all vectors of posterior beliefs are feasible

- The more signals to be interpreted, the more belief manipulability
- ► The more uniform the prior, the more belief manipulability The minimal prior across states is the lower bound for the maximal fit for any updated posteriors starting from given prior, i.e.,  $\bar{\delta}(\mu_s)^{-1} \leq \min_{\omega \in \Omega} \mu_0(\omega)$  for any  $\mu_s$

#### Corollary

If  $|S| \ge |\Omega|$  and  $\mu_0(\omega) = \frac{1}{|\Omega|}$  for every  $\omega \in \Omega$ , all vectors of posterior beliefs are feasible

### Comparative Statics, Binary Case

Let  $\mu_{0,\varepsilon} = \left(\frac{1}{2} - \varepsilon, \frac{1}{2} + \varepsilon\right)$  and  $\mathcal{F}_{\varepsilon}$  the set of the feasible vectors of posteriors with respect to the prior  $\mu_{0,\varepsilon}$ 

#### Proposition

For  $\varepsilon' < \varepsilon''$ , it holds that  $\mathcal{F}_{\varepsilon''} \subseteq \mathcal{F}_{\varepsilon'}$ 

 The closer the prior is to 50-50, the more belief manipulability



#### Sender's Problem



- ▶ The sender knows the receiver's preferences and the true model *t* 
  - Predictive probabilities of each signal realization  $Pr^{t}(s)$
  - Posterior induced by t conditional on each signal realization  $\mu_s^t$

Sender's Value of  $\mu$ , calculated over signal and state realizations using model t

$$V(\mu) = \sum_{s \in S} \Pr^t(s) \mathbb{E} \left[ U^S(a^*_s, \omega) 
ight]$$

The sender chooses the set of models  $M^*$  that maximizes his value at the resulting vectors of posterior beliefs

$$M^* \in rg\max_{M \subseteq \mathcal{M}} V(\mu^M)$$
 with  $\mu^M = \left(\mu_s^{m^*_s}
ight)_{s \in S}$ 

The fair model is adopted if

$$\operatorname{Pr}^{F}(W) > \operatorname{Pr}^{C}(W)$$

Calculate for which prior p this is the case:

$$p \,\, 99\% + (1-
ho) \,\, 1\% \geq p \,\, 1\% + (1-
ho) \,\, 50\%$$
 $p \geq 33\%$ 

### Inevitable Polarization, Binary Case

## Conflicting Models m, m' if $\pi^m(s_1|\omega_1) > \pi^m(s_1|\omega_2)$ and $\pi^{m'}(s_1|\omega_2) > \pi^{m'}(s_1|\omega_1)$

#### Proposition

For each pair of conflicting models, there exists a threshold p such that for every s1.  $\mu_s(\omega_1) < \mu_0(\omega_1)$  if  $\mu_0(\omega_1) < p$ 2.  $\mu_s(\omega_1) > \mu_0(\omega_1)$  if  $\mu_0(\omega_1) > p$ 

Any pair of conflicting models induces a Bayes-inconsistent vector of posteriors

The prior drives the direction of polarization

## Stylized Fact 1: Confidence in Fair Elections Persily and Stewart (2021)



## Stylized Fact 2: Confidence in Vote Count by State Persily and Stewart (2021)



#### Set of Feasible Vectors of Posteriors: Default Model

The receiver holds a default model d, known by the sender

- Holding a default model restricts belief manipulability
- With a default model, the sender can attain the same outcome with ex-ante or ex-post communication of models (Schwartzstein & Sunderam, 2021)



Back

The set of the feasible vectors of posteriors without the default model is the union of the sets of the feasible vectors of posteriors for every default model

