# Weighting Competing Models

Chiara Aina<sup>\*</sup> Florian H. Schneider<sup>†</sup>

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#### Abstract

We study how individuals update their beliefs in the presence of competing datagenerating processes, or models, that could explain observed data. Through experiments, we identify the weights participants assign to different models and find that the most common updating rule gives full weight to the model that best fits the data. While some participants assign positive weights to multiple models consistent with Bayesian updating—they often do so in a systematically biased manner. Moreover, these biases in model weighting frequently lead participants to become more certain about a state regardless of the data, violating a core property of Bayesian updating.

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<sup>\*</sup>Universitat Pompeu Fabra and Barcelona School of Economics. E-mail: chiara.aina@upf.edu.

<sup>&</sup>lt;sup>†</sup>Department of Economics and Center for Economic Behavior and Inequality (CEBI), University of Copenhagen. E-mail: flsc@econ.ku.dk.

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# 1 Introduction

Economic decision-making often occurs in situations where individuals have to interpret new information to form expectations about relevant outcomes. A large body of literature investigates how people revise beliefs in response to new information, focusing on situations where a single data-generating process, or *model*, provides a clear interpretation of the data. However, in many contexts, individuals are faced with multiple, conflicting interpretations, making it unclear how the data have been generated and how they should be interpreted.

For example, in asset markets, investors often use data on past performance to revise beliefs about future price trends but may encounter conflicting models to predict future returns based on historical returns. Such models might include mean reversion—"the asset's price went up in the past, so it will go down in the future"—, momentum—"the asset's price went up in the past, so it will continue to rise"—, or random walk—"past price movement provides no information about future performance" (e.g., Barberis et al., 1998). Similarly, in politics, contrasting views about election integrity, such as "vote counting is unbiased" versus "elections are rigged," lead to different beliefs about the legitimate winner of an election based on the same reported election result. In healthcare, debates about the safety of medical interventions arise because of contrasting views on whether "vaccine tests are trustworthy" or "vaccine tests are unreliable." Despite the prevalence of settings where people encounter conflicting models, little is known about how they update their beliefs in such situations.

This paper uses experiments to study how individuals update their beliefs when confronted with conflicting models that could explain the observed data. As in the examples above, we focus on a setting where participants aim to learn about an unknown state of the world—such as future asset returns, the legitimate winner of an election, or vaccine safety—based on a potentially informative yet noisy signal. Models provide distinct interpretations of the signal, proposing a data-generating process that quantifies the link between the signals and the states. Participants are presented with multiple models that may have generated the signal and make opposing predictions about the true state given the observed signal. We then analyze how individuals weight the models to aggregate the corresponding predictions about the true state. Specifically, we study which potentially biased updating rules—introduced below—describe how they weight models.

Our findings offer an empirical basis for a rapidly growing body of theoretical literature that formalizes narratives as models, as introduced by Schwartzstein and Sunderam (2021). Papers within this literature assume *model selection*, where agents form predictions about the state by assigning full weight to a single model. This relates to the tendency to simplify Bayesian reasoning into categorical thinking (Mullainathan, 2002; Mullainathan et al., 2008). Among plausible criteria for model selection, the most common assumption is that agents adopt the model that best fits the data, which we refer to as *model selection via maximum likelihood*. This approach is also considered in the literature on ambiguity aversion (e.g., Dempster, 1967; Shafer, 1976; Gilboa and Schmeidler, 1993). Notably, placing full weight on a single model contrasts sharply with Bayesian updating: to revise their beliefs, a Bayesian decision maker retains multiple model predictions and appropriately weights them by their likelihood of being the true data-generating process given the observed signal. Alternatively, agents might assign positive weight to multiple models but in a biased manner. Our experiment allows us to differentiate between these various rules and examine their implications for belief updating.

Studying belief updating with naturally occurring data poses the key challenge that the models individuals might consider are unknown, making it difficult to distinguish between the predictions of different updating rules. To overcome this challenge, we design a novel experimental paradigm that allows us to construct and control the models that may have generated the data. Our approach builds on the classic balls-and-urns updating task (Edwards, 1968) and closely aligns with the theoretical papers that formalize narratives as models. Participants observe a signal and are tasked with forming accurate beliefs about an unknown state. Unlike the classical task where participants encounter only one model, our experiment presents participants with two models, one of which is randomly selected as the true data-generating process responsible for producing the observed signal. We elicit participants' beliefs about the state in a series of seven of these updating tasks featuring different model pairs to test the robustness of our results and to assess participants' consistency in applying updating rules. Additionally, each participant encounters some model pairs twice, allowing us to further distinguish between different updating rules, study important consequences of such biases in model weighting, and assess the role of stochasticity in the use of updating rules.

To investigate biases in model weighting, it is crucial to distinguish these from other biases in belief updating, especially those related to deriving the model predictions from the observed signal. We address this by providing participants with the Bayesian prediction for each model upon observing the signal. This approach eliminates biases in deriving model predictions—well-documented in the literature—enabling us to identify model weights and classify reported beliefs according to different updating rules. Providing the model predictions mirrors a realistic feature of many real-world settings where individuals encounter a model alongside its prediction. For example, in asset markets, a financial advisor might advise an investor to buy an asset that has recently risen in price, explaining the concept of "momentum," along with the prediction that the asset's price will continue to rise in the future. Or, a politician who lost an election might tell voters that the "elections were rigged," and thus, that they are the rightful winner.

We find that the most frequent updating rule in our data is selecting the best-fitting model, which accounts for one-third of all elicited beliefs. This result provides strong evidence for model selection via maximum likelihood. Another twelve percent of beliefs correspond to selecting the worst-fitting model. What could lead participants to select the worst-fitting model? This could be attributed to occasional errors in identifying the best-fitting model, resulting in a stochastic version of model selection via maximum likelihood, or to the use of alternative criteria for model selection. We explore two plausible alternative criteria: participants might choose models either based on their *informativeness* given the observed signal (inspired by Yang, 2023) or based on characteristics unrelated to the signal, which we refer to as *dogmatic model selection*, a concept related to common assumptions in the literature on learning with misspecified models (see the discussion in Ba, 2024). Unlike the maximum likelihood criterion, these alternatives have limited predictive power. Instead, our findings predominantly support a stochastic version of model selection via maximum likelihood. This is further confirmed by a simple calibration exercise: model selection in our data is well-described by a stochastic version of model selection via maximum likelihood, with only a small minority of participants employing other model selection criteria.

Model selection is a widespread phenomenon in our data: nearly half of participants' beliefs are consistent with using only a single model to update beliefs. However, a substantial portion of participants assign positive weights to both models, albeit in a biased way. Only a small percentage of guesses (8%) align with Bayesian updating. Instead, a larger share (18%) is consistent with an extreme form of underinference about the models, where participants weight the model predictions by the prior rather than by the posterior over the models. We refer to this as *one-stage updating*: participants recognize that the signal is informative for learning about the state given each model but fail to recognize that the signal is also informative about the underlying model, resulting in an improper combination of the model predictions. Such an updating rule is assumed in the literature on persuasion with coarse thinking (Mullainathan et al., 2008). One-stage updating is the second most common rule in our data, following model selection via maximum likelihood, which represents an extreme form of overinference about the models.

Notably, model selection, Bayesian updating, and one-stage updating can account for over 70% of all elicited beliefs. Hence, these few rules capture the participants' beliefupdating process to a large extent. We also explore alternative updating rules, and find that some other participants employ less extreme forms of over- or underinference about the models. Our analysis finds no evidence that any other rules are applied frequently or consistently in our data.

Participants display a high degree of consistency in applying these updating rules. We show this with two approaches. First, we analyze the updating tasks in which participants face the same model pairs twice but observe different signal realizations. In these data, we can examine the posterior beliefs reported by participants for both signals, which we refer to as *vectors of posteriors*. The most frequently reported vector of posteriors corresponds to selecting the best-fitting model for both signal realizations, followed by the one predicted by one-stage updating. Second, we investigate whether participants follow the same updating rules in all seven updating tasks. We find that more than 40% of the participants use the same rule for all seven guesses, 49% when allowing to deviate from

a rule in one task, and 60% when requiring only a majority of consistent guesses. Thus, most participants use one of the updating rules consistently throughout the experiment.

Importantly, the data on the reported vectors of posteriors also allow us to test a key prediction from the literature on model persuasion: when presented with conflicting models, individuals may become more certain about a state, regardless of the observed data. We refer to such beliefs as *Bayes-inconsistent*, as they violate a basic property of Bayesian updating. When participants encounter conflicting models and consistently select the best-fitting model, they are predicted to hold Bayes-inconsistent beliefs (Aina, 2024). Our data reveal that half of the reported vectors of posteriors are Bayes-inconsistent, where the reported guesses for both signals are both either higher or lower than the prior. This widespread emergence of Bayes-inconsistencies underscores both the relevance and the power of persuasion using conflicting models and contrasts with classic persuasion theories, such as Bayesian persuasion (Kamenica and Gentzkow, 2011). Moreover, these inconsistencies exhibit a pattern aligned with confirmation bias, which is the tendency to confirm one's beliefs (e.g., Rabin and Schrag, 1999; Fryer et al., 2019).<sup>1</sup> While in previous studies this pattern can be attributed to motivated beliefs or strong initial beliefs, we find that Bayes-inconsistent beliefs can also arise in neutral settings—where individuals have neither a personal stake nor a preexisting stance—due to the presence of conflicting models and biases in model weighting.

To study the generalizability of our findings, we conduct an additional study where we independently vary two features of the updating environment and examine whether model selection remains prevalent. First, we examine a setting where participants are not given an objective prior over the models. This allows us to provide insights into situations where individuals encounter multiple models without an objective distribution over them, a setting that is also relevant to the literature on ambiguous beliefs. Second, we investigate a setting where model predictions are not immediately available, requiring participants to exert some effort to reveal each of them. This endogenous acquisition of model predictions helps us shed light on settings where these predictions are not readily available, and individuals need to take deliberate action to access them, reflecting search costs, cognitive effort, or attentional constraints. Our data show that participants deliberately seek model predictions. In only in 6% of updating tasks, participants proceed without acquiring any predictions, while in 10% of tasks, participants reveal the prediction of one model typically the best-fitting one. In the remaining 84% of tasks, they reveal predictions for both models. Moreover, we find that the updating patterns documented in our main study extend to both these updating environments. Compared to a baseline condition that replicates our main study, neither treatment condition leads to meaningful changes

<sup>&</sup>lt;sup>1</sup>Lord et al. (1979) and subsequent studies instructed participants to review information on controversial topics, assess whether it supported or opposed the issue, and report any shifts in their beliefs. These studies consistently found that participants' posterior attitudes became more favorable if they were initially supportive or more opposed if they were initially critical. This suggests that individuals may adjust their beliefs in a preferred direction, regardless of the evidence, exhibiting a pattern closely related to Bayes-inconsistent beliefs.

in updating behavior. Model selection is equally prevalent in all treatment conditions, ranging from 55% to 57% guesses consistent with selecting one model. In line with our main study, the most frequent updating rule is selecting the best-fitting model, with a frequency of 44% in all treatments, followed by one-stage updating, with a frequency ranging from 12% to 16%.

In summary, we find evidence supporting the theoretical assumption that, when faced with competing models, individuals tend to adopt only the model that best fits the observed data for updating their beliefs. This updating rule is prevalent across different environments, including those with and without a specified model prior. We also uncover other biases in model weighting, particularly one-stage updating, and mistakes in identifying the best-fitting model. Finally, we show that a large share of vectors of posteriors are Bayes-inconsistent, which fundamentally violates Bayesian updating and supports a key prediction from the theoretical literature on model persuasion. These findings not only provide empirical foundations for the assumption in recent papers formalizing narratives as models but also hold important implications for policy-oriented research, which we address in the conclusion.

This paper contributes to the fast-growing literature on narratives in economics, started by Shiller (2017). While there is no consensus in economics on the term narrative (see Barron and Fries, 2024b), narratives are mostly formalized in two ways: directed acyclical graphs (DAGs; e.g., Eliaz and Spiegler, 2020, 2024; Eliaz et al., 2021, 2022) or models as introduced by Schwartzstein and Sunderam (2021). Several papers build on the latter framework, and either follow Schwartzstein and Sunderam (2021) by assuming that agents select the best-fitting model (Ichihashi and Meng, 2021; Aina, 2024; Jain, 2024; Schwartzstein and Sunderam, 2024; Bauch and Foerster, 2024), or consider alternative updating rules (Yang, 2023; Ispano, 2024; Wojtowicz, 2024).<sup>2</sup> Building on these assumptions, these papers then study how a persuader, whose preferences might be misaligned with those of a receiver, can use models to manipulate the receiver's beliefs (model persuasion). Our study builds on this framework yet eliminates any strategic considerations. This approach enables us to isolate biases in model weighting from strategic factors, including skepticism and mistrust towards models provided by persuaders.

The empirical literature on narratives has thrived in the last years (e.g., Andre et al.,

<sup>&</sup>lt;sup>2</sup>The maximum likelihood criterion is also employed to study a wide range of settings, such as how individuals combine multiple forecasts (Levy and Razin, 2021), or how they face frictions in evaluating the model's fit to data due to computational and cognitive constraints (Samuelson and Steiner, 2024), or how to shift paradigm upon observing an unlikely news (Ortoleva, 2012). In the same spirit, Izzo et al. (2023) formalize models as linear relations between policies and their outcome, assuming that the model with the smallest mean squared error is adopted. By contrast, Barberis et al. (1998) posit that individuals hold competing mental models of asset markets and update beliefs about these models based on stock market signals, following Bayesian updating. This aligns with the literature of psychology which increasingly models cognition as probabilistic inference, often through a Bayesian framework (Tenenbaum et al., 2011; Chater and Oaksford, 2008, for overviews), and justified through the idea of sampling even within individuals (Vul and Pashler, 2008; Vul et al., 2014). Instead, other studies suggest that individuals tend to select the hypothesis with the highest explanatory power (e.g., Douven and Schupbach, 2015a,b).

2022, 2023; Graeber et al., 2024a,b; Bursztyn et al., 2023), with some recent experimental studies exploring related questions tied to the formalized notions of narratives. Barron and Fries (2024a) demonstrate in a financial setting that persuaders take model fit into account when proposing a model, and that the fit of the model is important for its persuasiveness. In contrast, we focus on how individuals update their beliefs when confronted with multiple models, distinguishing between different updating rules. Two experimental papers building on DAGs answer questions that relate to ours: Ambuehl and Thysen (2024) analyze the determinants of the selection of causal structures, while Kendall and Charles (2022) examine the effectiveness of causal structures in influencing decisions, uncovering averaging behaviors in the presence of multiple DAGs. In these experiments, participants are provided with rich datasets that maintain fixed correlations between variables, along with one or more DAGs that describe the potential causal relationships among these variables and imply different optimal actions. In contrast, our focus is on belief updating, which depends on the correlations between variables rather than on their causal links. Thus, we present participants with different models that propose different correlations between the variables, keeping the causal structure fixed.<sup>3</sup>

Our paper lies at the intersection of research on narratives and belief updating, thereby also making contributions to the latter. A large and long-standing literature documents biases in belief updating when individuals encounter a single model (for reviews, see Benjamin, 2019; Camerer, 1995). Recent papers have highlighted how features of the information structures (Ba et al., 2022; Augenblick et al., 2021) and the environment (Bordalo et al., 2023) might impact the type of biases observed in belief updating, such as over- and underinference or base-rate neglect. The paper most closely related to ours is Liang (2022), which compares belief updating across two settings: individuals are presented with either two data-generating processes that differ in accuracy (uncertain news) or an equivalent data-generating process (certain news) leading to the same Bayesian prediction. His findings indicate a tendency to underreact in the first setting compared to the second. Rather than focusing on the comparison between uncertain and certain news, our contribution is to study biases in model weighting and their implications for belief updating. We do so by designing an experiment where we can identify the model weights and thus classify participants' behavior into different updating rules.

The rest of the paper is organized as follows: Section 2 presents the conceptual framework, Section 3 describes our experimental design, Section 4 presents the results from our analysis, and Section 5 discusses the results and concludes.

<sup>&</sup>lt;sup>3</sup>Relatedly, Fréchette et al. (2024) and Kendall and Oprea (2024) study how people form and extract mental models for these large datasets in the absence of provided DAGs. Fréchette et al. (2024) finds that the recurring mistakes are associated with ignoring or misunderstanding correlations in the data.

## 2 Framework

This section outlines the theoretical framework for understanding how individuals update their beliefs when confronted with conflicting models. Building closely on the theoretical framework of model persuasion, we outline the main updating rules under examination.

Consider a decision-maker (DM, hereafter) with the goal of learning about an unknown binary state of the world,  $\omega \in \{A, B\}$ , where both states are initially equally likely. The DM observes a signal,  $s \in \{p, o\}$ , and revises their beliefs about the state. There are two competing models, Model 1  $(m_1)$  and Model 2  $(m_2)$ , that could explain the signal and provide predictions about the state. More precisely, each model specifies the probability of each signal conditional on each state,  $\Pr^m(s|A)$  and  $\Pr^m(s|B)$ . By applying Bayes' rule, a model makes predictions about the state for each signal,  $\Pr^m(A|s)$ .<sup>4</sup>

The DM can then use the observed signal to infer (1) how likely the state is given each model and (2) how likely the models are. The resulting posterior attached to state A conditional on observing signal s can be expressed as a weighted average of the two model predictions over state A:

$$\hat{\Pr}(A|s) = \rho_s^{m_1} \Pr^{m_1}(A|s) + \rho_s^{m_2} \Pr^{m_2}(A|s),$$
(1)

with  $\rho_s^{m_1}, \rho_s^{m_2} \in [0, 1]$  and  $\rho_s^{m_1} + \rho_s^{m_2} = 1$ . The weight  $\rho_s^m$  corresponds to the weight assigned to model *m* given signal *s*.

A large and long-standing literature has focused on how people update beliefs about states when exposed to one model, which relates to the matter of correctly forming the model predictions,  $\Pr^m(A|s)$ . In contrast, this paper focuses on how people learn about the models and, in turn, weight the model predictions,  $\rho_s^m$ , when making inference about the state. To study model weighting, we assume that the DM correctly derives the Bayesian model predictions. One interpretation of this assumption is that the DM encounters the models together with their predictions, which we consider a realistic feature of relevant settings. We explain in Section 3 how we ensure that this assumption is met in our experimental design.

A Bayesian DM derives the model weights by using Bayes' rule:  $\rho_s^m = \Pr(m|s) = \Pr(m)\Pr(s|m)/\Pr(s)$ , where  $\Pr(m)$  is the prior over models and  $\Pr(s|m)$  is the *likelihood* of observing signal *s* given model *m*, which we also refer to as the *fit* of model *m* given *s*. Such Bayesian model weights have been assumed, for example, by Barberis et al. (1998) to aggregate competing mental models of price movements in asset markets. Figure 1a illustrates the posteriors of a Bayesian DM. In this and similar figures, the axes represent posterior beliefs for the two signal realizations:  $\Pr(A|p)$  on the x-axis and

<sup>&</sup>lt;sup>4</sup>This framework can capture the examples discussed in the introduction. In the example about asset markets, the future performance of an asset could either be high ( $\omega = A$ ) or low ( $\omega = B$ ). The signal realizations include information about past performance (e.g., asset prices recently decreased: s = o). A model predicting mean reversion would imply  $\Pr^m(A|o) > \Pr^m(A)$ , whereas a model predicting momentum would imply  $\Pr^m(A|o) < \Pr^m(A)$ .

 $\Pr(A|o)$  on the y-axis. We refer to points in this graph as vectors of posteriors. Each model induces a vector of posteriors corresponding to the model predictions, represented by a blue diamond shape. The DM's prior over the state is represented by the black circle labeled "prior." The vector of posteriors for a Bayesian DM lies on the black line depending on their prior over models. We restrict our attention to the case where the models are initially equally likely,  $\Pr(m_1) = 50\%$ , represented by a purple square labeled "Bayesian."



Figure 1: Predictions for Vectors of Posteriors

Notes. The figures illustrate two models with the following parameters:  $\Pr^{m_1}(p|A) = 1/6$ ,  $\Pr^{m_1}(p|B) = 3/6$ ,  $\Pr^{m_2}(p|A) = 5/6$ , and  $\Pr^{m_2}(p|B) = 1/6$ . In both figures, the black circle corresponds to the prior over the state, the blue diamonds to the two model predictions, the purple square to Bayesian updating with  $\Pr(m_1) = 50\%$ , the red square to the one-stage updating, the blue square to selecting the best-fitting model, and the orange square to selecting the worst-fitting model; the gray areas represent the Bayes-consistent vectors of posteriors. The black line in Figure 1a illustrates the Bayesian vectors of posteriors for any prior over the models. In Figure 1b, colored areas represent the classification for over- and underinference about the models with the following color code: overinference for both signals (blue), underinference for both signals (red), overinference for a signal and wrong direction for the other signal (purple), wrong direction for both signals (yellow), overinference for a signal and wrong direction for the other signal (orange). The colored lines capture overinference (blue), underinference (red), and wrong direction (yellow) for a constant degree of bias in inference about the models,  $\alpha$ , as defined by  $\rho_s^m = \hat{\Pr}(m|s) = 1/(1 + (\Pr(s|m')/\Pr(s|m))^{\alpha})$  (see Appendix D.1 for details).

While Bayesian updating typically requires the DM to assign positive weight to both models, recent theoretical papers propose an alternative approach—which we call *model* selection—where individuals place full weight on one of the two models, i.e.,  $\rho_s^m \in \{0, 1\}$ . The most common assumption in the literature on model persuasion, as introduced by Schwartzstein and Sunderam (2021), proposes the DM selects the model that maximizes the likelihood of the data, which we refer to as model selection via maximum likelihood. Formally,  $\Pr(s|m) > \Pr(s|m')$  implies  $\rho_s^m = 1$  and  $\rho_s^{m'} = 0$ , and thus the DM's posterior

matches the prediction of the best-fitting model,  $\Pr(A|s) = \Pr^m(A|s)$ .<sup>5</sup> If the process of model selection exhibits some stochasticity, as suggested by Wojtowicz (2024), the DM may occasionally make mistakes in evaluating the fit levels of the competing models and select the worst-fitting model instead of the best-fitting one. Such mistakes differ from the standard way of modeling errors in reported guesses, which posits that DMs make frequent but mostly minor mistakes. In contrast, should errors in model selection occur, they would be of much larger magnitude.

Our main focus is on model selection via maximum likelihood because of its prevalence in theoretical work, but we also consider other criteria for model selection. First, a DM may consistently select the same model irrespective of the observed signal, as commonly assumed in the literature on learning with misspecified models (see the discussion in Ba, 2024). This would be the case if the DM applies a model selection criterion based on features of the models that are independent of the signal, such as symmetry or simplicity. Borrowing the term from Ba (2024), we refer to such rules as *dogmatic model selection*. A further alternative is *model selection based on informativeness*. Inspired by Yang (2023), this rule prescribes that the DM selects the most informative model given the observed signal, that is, the model with the posterior over the states closest to 100% or 0%.<sup>6</sup>

Biases in model weighting may not be limited to model selection; a DM that assigns positive weights to both models may do so in a systematically biased manner. In particular, the DM may exhibit over- or underinference about the models. Note that a Bayesian DM also revises their beliefs about the best-fitting model m upwards, i.e.,  $\Pr(m|s) > \Pr(m) = 50\%$ . Therefore, any  $\rho_s^m \in (\Pr(m|s), 1]$  would reveal overinference about the models: the DM overreacts to the observed signal, expecting model m to be the true data-generating process with an excessively high probability. With  $\rho_s^m = 1$ , model selection via maximum likelihood corresponds to the extreme form of overinference about the models. Conversely, a DM may exhibit underinference about the models if  $\rho_s^m \in [0.5, \Pr(m|s))$ . In its extreme form, the DM weights the two model predictions using the prior over the models,  $\rho_s^m = \Pr(m) = 50\%$ . We refer to this as one-stage updating because the DM recognizes that the signal is informative for learning about the state given each model but fails to update beliefs about the models, improperly combining the model predictions. Such an updating rule is assumed by Mullainathan et al. (2008) to study persuasion with categorical thinking. To capture more broadly over- and underinference about the models, we present in Appendix D.1 a theoretical framework that can account for less extreme forms of over- and underinference about the models.

<sup>&</sup>lt;sup>5</sup>This approach is equivalent to selecting models according to the Bayes factor, i.e., the ratio of the fit levels of the two competing models  $\Pr(s|m)/\Pr(s|m')$ . It quantifies the support for one model over the other: if higher than 1, the data supports model m more strongly than m'.

<sup>&</sup>lt;sup>6</sup>Yang (2023) proposes model selection based on decisiveness: individuals choose models that provide clear guidance for optimal action, thus favoring extreme models. This criterion relies on the DM's incentives; thus, decisiveness and informativeness can differ for some payoff structures. Our experiment is not designed to study model selection based on decisiveness, as participants do not make subsequent decisions, and incentives are chosen only to incentivize truthful belief elicitation.

Figure 1b illustrates the predictions for the different biases in model weighting. The squares labeled "Bayesian" and "one-stage" correspond to the predictions of Bayesian and one-stage updating. A DM exhibiting overinference (underinference) about the models for both signals would report a vector of posteriors in the blue (red) area. The blue square labeled "best-fitting" corresponds to the prediction of model selection via maximum likelihood. If the DM makes mistakes in applying this rule, their reported vectors of posteriors may coincide with the predictions of Model 1 ("m1"), Model 2 ("m2") or the worst-fitting model ("worst-fitting"). A vector of posteriors at "m1" or "m2" might also indicate dogmatic model selection, while model selection based on informativeness would imply reporting "m2."

Biases in model weighting can lead to patterns that are highly inconsistent with Bayesian updating. For instance, the DM might become more confident in one state regardless of the observed signal, as illustrated by the "best-fitting" prediction in Figure 1b. We say the reported vector of posteriors is *Bayes-consistent* if, for all  $s \neq s'$ , (i)  $\hat{\Pr}(A|s) > \Pr(A)$  and  $\hat{\Pr}(A|s') < \Pr(A)$ , or (ii)  $\hat{\Pr}(A|s) = \hat{\Pr}(A|s') = \Pr(A)$ . Bayes consistency is the sole restriction that Bayesian updating generally imposes on vectors of posteriors, irrespective of the set of plausible models the DM may entertain (Aina, 2024).<sup>7</sup> However, for model selection via maximum likelihood, each signal triggers the adoption of a different model, and, as a result, *Bayes-inconsistencies* occur every time the DM is confronted with conflicting models, that is, pairs of models for which  $\Pr^m(s|A) > \Pr^m(s|B)$  and  $\Pr^{m'}(s|A) < \Pr^{m'}(s|B)$ . In Figure 1, the gray areas correspond to Bayes-consistent vectors of posteriors, while the white areas highlight the Bayes-inconsistent ones. The two illustrated models are conflicting as their predictions fall on opposing gray areas and, indeed, the best-fitting prediction lies in the white area.

Bayes-inconsistencies do not only result from model selection via maximum likelihood. Rather, they occur frequently when DMs are faced with competing models and exhibit strong overinference about the models. In contrast, when a DM entertains only a single model, there is no reason to expect Bayes-inconsistencies to arise. This insight is also important for distinguishing model weighting from another possible updating procedure: instead of weighting the two model predictions, the DM could aggregate the models into a single model and then update their beliefs using this model. According to this approach, which we discuss in detail in Appendix D.3, the resulting posterior beliefs would be Bayes-consistent—even if the DM's belief revision exhibits over- and underinference to the signal—since they are derived using a single model. Therefore, Bayes-inconsistencies offer compelling evidence of novel biases in belief updating that arise when confronted with multiple models. Moreover, these inconsistencies highlight that biases in model weighting can lead to deviations from the Bayesian benchmark that fundamentally differ

<sup>&</sup>lt;sup>7</sup>Following Aina (2024), Bayes-consistency requires that the prior can be expressed as a convex combination of the posteriors across signals for each state; see Shmaya and Yariv (2016) and Bohren and Hauser (2024) for analogous results. Note that Bayes-consistency is a weaker condition than requiring a distribution of posteriors to be Bayes-plausible as in Kamenica and Gentzkow (2011). In Figure 1, the only vector of posteriors consistent with Bayes-plausibility is the Bayesian prediction.

from those observed when updating with a single model.

This section highlights that the literature has considered a wide range of plausible model weighting rules. Distinguishing between these rules has important implications. For example, Bayesian inconsistencies only arise from rules that result in strong overinference about the models. This underscores the need for empirical evidence to determine how individuals actually update in these situations, which is the purpose of our study. We examine the descriptive validity of Bayesian updating, one-stage updating, and model selection using different selection criteria. Specifically, we investigate which rules are applied frequently and whether individuals apply them consistently across updating tasks. Finally, we also study whether Bayesian inconsistencies occur frequently.

# 3 Experimental Design

The focus of our experiment is to examine how people update beliefs in the presence of competing models. To address this question and comprehensively study biases in model weighting, our experimental design meets the following criteria: (i) using a belief updating task where participants can be exposed to multiple models that could have generated the data and that closely follows the theoretical framework; (ii) controlling the true data-generating process and the pairs of models participants encounter; (iii) avoiding biases and errors in deriving models' predictions; and (iv) collecting the vectors of posteriors.

To achieve features (i) and (ii), we build on the classic "balls-and-urns" paradigm (Edwards, 1968). Regarding feature (iii), we provide participants with the model predictions, as detailed below. This approach eliminates biases in deriving model predictions, enabling us to directly measure biases in model weighting. Without this feature, identifying biases in model weighting would be challenging. This relates to Bohren and Hauser (2024), which highlights the challenges of distinguishing biases from non-Bayesian updating and those from the use of misspecified models. An alternative approach would be to directly elicit model weights rather than inferring them from posteriors over the state, but this would fundamentally alter the updating environment that motivated our study.<sup>8</sup> To implement feature (iv), participants encounter some updating tasks twice, allowing us to recover the vectors of posteriors. This is important for gaining deeper insights into the updating process and for better distinguishing among potential biases in model weighting. Additionally, it allows us to test for the systematic presence of Bayes-inconsistencies, a key consequence of certain biases in model weighting.

In the classic balls-and-urns paradigm, the participant encounters one model. There are

<sup>&</sup>lt;sup>8</sup>We choose to elicit posteriors over the state because we want to closely mimic both the theoretical framework and the natural decision-making process, in which individuals use models as means to update their beliefs about the state in response to new information. Directly eliciting model weights would shift the focus to a setting where individuals are interested in learning about models themselves, thereby eliminating the central role of the payoff-relevant state of the world—a substantially different decision-making context. Moreover, we believe that directly eliciting model weights may communicate to participants that we expect them to react to the signal in a particular way.

two bags, which represent the state of the world,  $\omega \in \{A, B\}$ , and each contains a number of colored balls. In our experiment, bags always have a total of six balls, with each ball being either purple, p, or orange, o. A model m sets the number of purple balls in each bag,  $\Pr^m(p|A)$  and  $\Pr^m(p|B)$ . The participants know the composition of both bags given the model. In the task, one of these bags is randomly selected. The participants do not know which bag is selected but observe the color of a ball drawn from the selected bag, that is, the signal  $s \in \{p, o\}$ . After observing the signal, the participants report their beliefs about the probability that bag A was selected.

To study model weighting, we introduce a novel updating task that closely follows the theoretical framework, building on the classic balls-and-urns paradigm. Before detailing the task, we outline the key idea. In this novel task, participants face two models. Participants learn the compositions of the bags according to both possible models before they observe the signal, as in Aina (2024). In this updating task, one of the two models is randomly selected to be the true data-generating process and to generate the observed signal. That is, the selected model dictates the composition of the balls in the bags. The remaining sequence follows the standard updating task: a bag is selected, and then a ball is drawn from that bag. The participants know neither the selected model nor the selected bag. After observing the color of the drawn ball, the participants are asked to report their beliefs about bag A being selected.

To introduce this novel setting to participants, we divide our experiment into three parts, progressively incorporating new elements through a series of updating tasks. We first introduce updating given a single model (Part 1), then the model predictions (Part 2), and, finally, updating with two models (Part 3). This helps participants to better understand the different elements of our novel updating task. Three details regarding the implementation of the tasks are worth mentioning. First, we use an animated interface illustrating the random draw of the state, the draw of the signal, and, in Part 3, the draw of the model, to recreate a realistic setting online and intuitively remind the participant of the task's basic structure. Second, we framed models as "robots" placing a specific number of orange and purple balls in each bag. The goal is to make the task more intuitive, especially in Part 3, where the participants encounter two models. Third, participants report their guesses using a slider. Next, we provide a more detailed discussion of each of the three parts.

Part 1 employs classic balls-and-urns updating tasks with one model. The purpose of this first part is to introduce and explain updating in the context of a single model. Participants make guesses for two different models, described in Table 1. Figure 2a illustrates the survey interface.

Part 2 mirrors the first part with one variation: the robot provides the participant with the correct Bayesian predictions,  $Pr^m(A|s)$ , given the observed ball. This part introduces and explains model predictions. Predictions are conveyed both verbally and through a colorful pin on the slider. Participants then make two guesses with the same models as



Figure 2: Experimental Interface

in Part 1. Since the correct Bayesian prediction are provided, these updating tasks only serve as a learning exercise.

Part 3, which is the focus of our study, alters the paradigm by introducing two robots, each proposing a different distribution of colored balls in the bags. With equal probability, one of the two robots is chosen to implement its proposed distribution. Hence, participants are presented with two models. Similar to the second part, each robot provides the Bayesian prediction given the observed drawn ball. Figure 2b illustrates the survey interface (see Appendix G for more details).

In Part 3, participants complete seven updating tasks. We carefully chose a total of five pairs of models, presented in Table 1. When choosing the model pairs, we focused on those that (i) have different fit levels, ensuring a single best-fitting model, (ii) are conflicting, so that model selection via maximum likelihood predicts Bayes-inconsistent vectors of posteriors, and (iii) allow us to distinguish between the different updating rules discussed in Section 2, (iv) exhibit diverse characteristics to assess the robustness of our findings. Appendix Figure B1 shows the predictions of the different updating rules for all model pairs. Moreover, Model Pair 3 and Model Pair 5 share a common model, which allows us to test, on aggregate, whether reported posteriors are reactive to making a model prediction more informative.<sup>9</sup>

Participants encounter these five model pairs in a random order, followed by a repetition of two model pairs (Model Pair 2 and Model Pair 3), also presented in a random order

<sup>&</sup>lt;sup>9</sup>Model Pair 4 is designed to compare two particular models: one fully uninformative (same number of purple balls in each bag) and one fully informative (only one color of balls in each bag). This pair offers a particularly strong test of model selection based on informativeness, which would predict a systematically higher weight towards the fully informative model regardless of the observed signal.

(see also Table 1, Column "Repeated"). We use the data from the repeated model pairs in two ways. First, when participants report posteriors for the same model pair observing different signals, we observe their resulting vector of posteriors,  $\hat{\Pr}(A|p)$  and  $\hat{\Pr}(A|o)$ .<sup>10</sup> Second, when participants report posteriors for the same model pair observing the same signals, we can use these observations to assess data quality by examining the consistency of reported guesses and to study stochasticity in applying model selection criteria.

Part	Pair	$\Pr^{m_1}(p A)$	$\Pr^{m_1}(p B)$	$\Pr^{m_2}(p A)$	$\Pr^{m_2}(p B)$	Repeated
1, 2		5/6	1/6			
1, 2		4/6	5/6			
3	1	1/6	2/6	5/6	2/6	No
3	2	1/6	3/6	5/6	1/6	Yes
3	3	6/6	1/6	1/6	2/6	Yes
3	4	1/6	1/6	0/6	6/6	No
3	5	4/6	2/6	1/6	2/6	No

#### Table 1: Models Used in the Experiment

Notes. This table describes all models used in the experiment. In Part 1 and Part 2, participants encounter a single model  $(m_1)$ , while in Part 3, they face two models  $(m_1 \text{ and } m_2)$ . In each updating task, there are two bags, representing the state of the world,  $\omega \in \{A, B\}$ , with each bag containing six balls, either purple (p) or orange (o). A model m sets the share of purple balls in each bag,  $\Pr^m(p|A)$  and  $\Pr^m(p|B)$ , as shown in the corresponding columns. Finally, in Part 3, participants encounter Model Pair 2 and Model Pair 3 twice, as indicated in the "Repeated" column.

We conclude the study by eliciting several survey items, including three contextualized updating tasks, a Berlin numeracy task (Cokely et al., 2012), a modified version of the Cognitive Reflection Test (Frederick, 2005), a selection of items to determine their thinking style (Keaton, 2017), and demographics.

**Incentives** The participants earned a completion fee of 6 USD and, depending on the accuracy of their guesses in the updating tasks, could earn a bonus of 2 USD. Only one of the eleven updating tasks was randomly selected for the bonus payment. Belief elicitation is incentivized using the binarized scoring rule (Hossain and Okui, 2013) and explained intuitively by following Danz et al. (2022).<sup>11</sup>

**Logistics and Sample** The experiment was pre-registered on AsPredicted and conducted on Prolific in April 2024, restricting the participant pool to US residents, aged 18-70 with approval rates of at least 95%.<sup>12</sup> The study was completed through a link to

 $<sup>^{10}</sup>$ We opt for repeated model pairs rather than eliciting vectors of posteriors directly. To do that, one would need to elicit beliefs with the strategy method; however, Aina et al. (2024) finds reported beliefs might differ if reported contingent on both signal realizations, without having observed the realized one.

<sup>&</sup>lt;sup>11</sup>Instructions clarify that "to maximize the chance of winning the bonus, it is in your best interest always to give a guess that you think is the true chance." A control question verifies the comprehension of this aspect. Participants have the option to review the details of the elicitation rule at their discretion.

<sup>&</sup>lt;sup>12</sup>The pre-registration plan is available at https://aspredicted.org/9YD\_HNH.

a Qualtrics survey, including instructions and control questions for each part (see Appendix G). A total of 300 participants completed the study successfully. The average payment was 7.5 USD, and the average duration was approximately 38 minutes. In our final sample, 51% are female, 31% have low schooling ("High school" or lower educational level), and the median age is 38.

## 4 Results

In this section, we analyze how individuals update beliefs when confronted with competing models. We analyze the data in multiple forms. First, we pool the data from all seven updating tasks and study individual guesses. Second, we investigate the reported vectors of posteriors to better distinguish between the predictions of different update rules and to test for the systematic presence of Bayes-inconsistencies. Third, we study whether participants consistently apply the same updating rules across all seven updating tasks. We begin by focusing on the main updating rules introduced in Section 2 and present results for additional potential updating rules in a subsequent section. Before presenting these analyses, we begin by evaluating the quality of the collected data.

### 4.1 Quality of Data

We consider two complementary methods to verify the quality of our data: assessing individual consistency when participants repeatedly face the same updating task and analyzing the aggregate reaction across model pairs.

**Consistency** First, we examine the consistency of guesses when participants face the same updating task. We focus on the repeated model pairs and analyze instances where participants observed the same signal twice.<sup>13</sup> This approach enables us to assess the noise in the reported guesses in our experiment. Our data reveal that a substantial portion of the guesses are consistent; specifically, 42% of these pairs of guesses are identical, and an additional 11% differ by only a small margin of at most 2 percentage points. The average distance between guesses is 11%, with a median of 1%. Appendix Figure A1 reports the cumulative distribution of this measure.

Across Model Pairs Second, we compare the reported guesses across two model pairs, Model Pair 3 and Model Pair 5, which share a common model. This approach allows us to test whether, on aggregate, participants are responsive to the information provided in our experiment by comparing their average beliefs across model pairs when making a model more informative for both signal realizations. If this were the case, we would

 $<sup>^{13}</sup>$ We can perform this test for a total of 300 pairs of guesses, corresponding to 220 participants: for 80 participants, different signals were drawn for both repeated model pairs, and we cannot perform this test; for 140 participants, we have the pair of guesses for one model; and for 80 participants, we have the pair of guesses for both model pairs.

expect that a DM reports a higher (lower) average guess when a purple (orange) ball is drawn in Model Pair 3 than in Model Pair 5 (see Appendix Figure A2).<sup>14</sup> Both these predictions are verified in our data. The average guesses given a purple ball are 67% for Model Pair 3 and 53% for Model Pair 5 (p-value < 0.001), and average guesses given an orange ball are 44% for Model Pair 3 and 50% for Model Pair 5 (p-value = 0.001).

The high consistency rate and aggregate reactions to changing a model across model pairs indicate high data quality and suggest that participants understand the choices they face.

### 4.2 Analysis of Individual Guesses

In this section, we analyze the individual reported guesses, pooling the data across all seven updating tasks. Figure 3 presents the distribution of the weights assigned to the best-fitting model,  $\rho$ , based on participants' guesses. The red distribution serves as a benchmark, illustrating how the weights would be distributed if participants adhered to Bayesian updating in all tasks. The blue distribution shows the recovered weights for the reported guesses that fall within the range of the two model predictions, which account for 87.86% of all guesses (N=1,845). The figure shows that the distribution of  $\rho$  deviates significantly from the Bayesian benchmark, with prominent peaks at 100%, 0%, and 50%, corresponding to the predictions of selecting the best-fitting model or the worst-fitting model, and one-stage updating, respectively. The absence of substantial peaks at other weights suggests that participants seldom employ alternative updating rules that would require a constant  $\rho$  across tasks.<sup>15</sup>

Interestingly, the mean reported guess is relatively close to the Bayesian benchmark, with an average deviation of 4.98 percentage points. While this difference is small, it is statistically significant (OLS regression with cluster-robust standard errors: t = 8.55, p < 0.001). Appendix Figure A3 plots the average reported guess alongside the predicted Bayesian prediction for each of the five model pairs and two signal realizations, revealing similar patterns. However, the belief distributions are multimodal, with few observations near the Bayesian prediction (see Figure 3 and Appendix Figures B2, B3 and B4). This highlights that while mean beliefs seem to align with Bayesian predictions, they fail to adequately capture the population's belief distributions—a pattern also emphasized in Bordalo et al. (2023).

We further classify the guesses by the point predictions of the different updating rules discussed in Section 2. Table 2 reports the percentage of guesses classified as Bayesian,

<sup>&</sup>lt;sup>14</sup>Such updating behavior is predicted by most of the updating rules that we consider, including Bayesian updating, one-stage updating, and model selection via maximum likelihood. Only dogmatic model selection does not make specific predictions in this setting, as it imposes no structure on model selection beyond the requirement that participants choose the same model for both signals.

<sup>&</sup>lt;sup>15</sup>We explore updating rules with constant model weights in Appendix D.2 and find little support for this behavior. We discuss these and other alternative updating rules in Section 4.5.





Notes. The figure plots the distribution of weights on the best-fitting model,  $\rho$ , across tasks. We report the implied weights for the reported guesses within the two model predictions in blue. We compute the implied weight as  $\rho = (guess - \Pr^{m'}(A|s))/(\Pr^m(A|s) - \Pr^{m'}(A|s))$ , where *m* represents the bestfitting model and *m'* the worst-fitting model given signal *s*. The red distribution serves as a benchmark, illustrating how the weights should be distributed if participants follow Bayesian updating in all tasks.

Type of Guess		Exact	Within 2 p.p.			
	%	95%-CI	%	95%-CI		
Bayesian	3.05	[1.48, 4.61]	8.14	[6.20, 10.09]		
One-stage	10.95	[8.21, 13.69]	18.00	[14.61, 21.39]		
Best-fitting	28.86	[25.03, 32.68]	33.10	[29.14, 37.05]		
Worst-fitting	10.95	[8.88, 13.03]	12.43	[10.26, 14.60]		
Within	34.05	[30.16, 37.94]				
Outside	12.14	[9.94,  14.35]				
Total	100.00		71.67			

#### Table 2: Classification of Guesses

*Notes.* Column "Exact" reports the shares of guesses that can be exactly classified as one of the point predictions (Bayesian, one-stage, best-fitting, and worst-fitting) or in one of the two residual categories ("Within" if the guess is within the two model predictions or "Outside" otherwise). Column "Within 2 p.p." reports the shares of guesses that fall within 2 p.p. around each point prediction. Note that 2.33% of reported guesses are classified both as Bayesian and one-stage, not included in the total. Columns "%" and "95%-CI" report the shares and corresponding 95% confidence intervals, using standard errors clustered at the individual level. We pool the data from all seven updating tasks.

one-stage, and model selection.<sup>16</sup> For model selection, we differentiate between selecting the best-fitting and the worst-fitting model. Strikingly, by only considering these four point predictions, we classify over half of all guesses when requiring an exact match to the predictions (Column "Exact") and more than 70% of guesses when allowing for a small deviation of at most 2 percentage points from the predictions (Column "Within 2 p.p."). In Column "Exact", we classify guesses that do not match any point prediction into two residual categories: "Within" if the guess falls within the model predictions and "Outside" otherwise. Reassuringly, only 12.14% of guesses are classified as outside, with only 8.33% deviating substantially from the model predictions (more than 2 percentage points). Among all outside guesses, 77% are extreme in the direction of the best-fitting prediction.<sup>17</sup>

The most frequent updating rule used in our data is model selection via maximum likelihood, accounting for about one-third of all guesses. The second and third largest categories are one-stage updating and selecting the worst-fitting model. Some guesses are consistent with Bayesian updating but these are less common; they account for about 8% of all guesses. The confidence intervals provided in Table 2 indicate that our estimates for the different shares are fairly precise. While Table 2 pools the data from all model pairs, Appendix Table B1 and B2 replicate this analysis for each model pair. As discussed in more detail in Appendix B, we find that the distributions of updating rules recovered from individual guesses are remarkably similar across different model pairs.

Overall, model selection is widespread and it accounts for 45.53% of reported guesses. The findings in Table 2 show that participants predominantly update their beliefs using the best-fitting model, providing evidence in support of model selection via maximum likelihood. But do participants also use other criteria to select models? To explore alternative criteria, we consider whether participants tend to select the most informative model. We find that around 18% of guesses align with model selection based on informativeness. Importantly, however, depending on the model pair, the predictions of the different model selection criteria overlap. Guesses that correspond to the most informative model are more frequent when the most informative model aligns with the best-fitting model (30%) than when it aligns with the worst-fitting model (11%).

To account for the overlaps in predictions, we regress an indicator of whether the participant selected Model 1 rather than Model 2 on model characteristics corresponding to different model selection criteria. For this analysis, we restrict our sample to observations corresponding to model selection. The results are presented in Table 3. Column

<sup>&</sup>lt;sup>16</sup>Appendix Table A1 replicates this classification for consistent guesses, as described in Section 4.1. The overall pattern closely resembles the one of Table 2, but with a higher share of guesses matching the prediction of the best-fitting model (48%) and less falling in the residual category "Within" (20%).

 $<sup>^{17}</sup>$ By adopting a broader classification of over- and underinference about the models, our analysis reveals that the most frequent bias is overinference, with 43% of guesses falling into this category. Furthermore, we find that 17% of guesses are classified as underinference about the models, while 24% represent inferences in the wrong direction. In particular, the residual category "Within" of Table 2 can be further divided into 17% underinference, 42% overinference, and 41% wrong direction.

Dependent Variable		Selects Model 1						
	(1)	(2)	(3)	(4)	(5)			
Best-fitting	0.450***		0.449***		0.446***			
	(0.041)		(0.043)		(0.043)			
Most Informative		$-0.151^{***}$	-0.005		-0.002			
		(0.039)	(0.035)		(0.080)			
				0 1 4 0 * * *	0 111***			
Model Pair 2				0.148	0.111			
				(0.048)	(0.041)			
Model Dair 2				0.000	0.040			
Model Fall 5				-0.009	0.049			
				(0.060)	(0.091)			
Model Pair 4				0.035	-0.011			
Model I all F				(0.066)	(0.061)			
				(0.000)	(0.004)			
Model Pair 5				-0.038	0.021			
				(0.065)	(0.077)			
				(0.000)	(0.011)			
Constant	$0.257^{***}$	$0.540^{***}$	0.259***	0.446***	0.214***			
	(0.027)	(0.023)	(0.034)	(0.045)	(0.041)			
Observations	836	836	836	836	836			
$\mathbb{R}^2$	0.203	0.021	0.203	0.020	0.211			

#### Table 3: Criteria for Model Selection

Notes. The table shows coefficient estimates from linear regressions of an indicator of selecting Model 1 on different explanatory variables. The sample consists of the 836 guesses that correspond to model selection (see Table 2). "Best-fitting" is an indicator for Model 1 having a higher fit than Model 2 given the observed signal. "Most Informative" is an indicator for Model 1 being more informative about the state than Model 2 given the observed signal. "Model Pair 2" to "Model Pair 5" are model pair fixed effects that capture any factors that are unconditional on the signal. Standard errors are clustered on the individual level (179 clusters) and are shown in parentheses. \* p < 0.10, \*\* p < 0.05, \*\*\* p < 0.01

(1) shows that participants are 45 percentage points more likely to select Model 1 when it is the best-fitting model, consistent with model selection via maximum likelihood. Columns (2) and (3) demonstrate that informativeness does not robustly predict model selection.<sup>18</sup> Finally, we investigate whether participants select models based on characteristics of models or of the model pair that are independent of the signal, thus aligned with dogmatic model selection. Columns (4) and (5) explore this by including model pair fixed effects. Most fixed effects coefficient estimates are small and not statistically significant, except for Model Pair 2, where participants seem to favor Model 1 regardless of the signal. However, the  $R^2$  changes minimally with the addition of these fixed effects, suggesting that the signal-independent characteristics do not play a major role in

<sup>&</sup>lt;sup>18</sup>As an additional test of model selection based on informativeness, we can restrict our attention to Model Pair 4, where participants encounter one fully informative (Model 1) and one fully uninformative model (Model 2). We do not observe participants to be systematically drawn to a specific model as shown in Column (4).

our setting. These results speak against a systematic form of dogmatic model selection, where most individuals consistently select the same model, but they do not rule out an unsystematic form of dogmatic model selection, where participants are drawn to different models in a balanced manner.

Therefore, our findings provide strong support for the importance of the maximum likelihood criterion, while offering little support for the informativeness and a systematic form of the dogmatic criteria, which exhibit little predictive power. Appendix Table A2 presents similar findings, allowing for a difference of at most 2 percentage points between guesses and predictions.

### 4.3 Analysis of Vectors of Posterior Beliefs

Next, we examine the reported vectors of posterior beliefs. We have data on the vectors of posteriors from the repeated model pairs, when participants observed different signals for the same pair of models. This data enables us to determine whether participants consistently use the same updating rule across different signals, to better distinguish different rules—especially for model selection—and to test for the presence of Bayesinconsistent vectors of posteriors.



Figure 4: Vectors of Posteriors

*Notes.* The size of the circles is relative to the number of observations. We pooled observations that are within 2 p.p. of the point predictions. Observations in the gray areas are Bayes-consistent, while observations in the white areas are Bayes-inconsistent.

Our analysis is based on a total of 300 vectors of posteriors (148 in Model Pair 2 and 152 in Model Pair 3), gathered from 220 participants.<sup>19</sup> Figure 4 plots the distribution of the reported vectors of posteriors. Table 4 categorizes these vectors of posteriors according to the point predictions discussed in Section 2, pooling together data from both model pairs.

<sup>&</sup>lt;sup>19</sup>For 80 participants, the same signal was drawn for both repeated model pairs, preventing us from recovering any vector of posterior beliefs. Note, however, that this attrition is purely random.

Type of Vector	Exact		Wi	ithin 2 p.p.	Within 5 p.p.		
	%	95%-CI	%	95%-CI	%	95%-CI	
Bayesian	2.67	[0.42, 4.91]	3.33	[0.92, 5.75]	5.33	[2.21, 8.46]	
One-stage	6.00	[2.88, 9.12]	9.67	[5.77, 13.56]	12.67	[8.24, 17.09]	
Best-fitting	19.67	[14.58, 24.76]	23.33	[17.87, 28.80]	24.67	[19.09,  30.24]	
Worst-fitting	2.33	[0.61,  4.06]	2.67	[0.83,  4.50]	2.67	[0.83,  4.50]	
Model 1	4.33	[1.87,  6.79]	5.33	[2.66, 8.01]	5.67	[2.93, 8.41]	
Model 2	4.67	[1.96, 7.37]	6.00	[3.04, 8.96]	6.00	[3.04, 8.96]	
Prior	2.00	[0.00,  4.05]	2.00	[0.00,  4.05]	2.00	[0.00,  4.05]	
Total	41.67		52.33		59.01		

 Table 4: Classification of Vectors of Posteriors

*Notes.* The table reports the shares of vectors of posteriors that have an Euclidean distance from the prediction vector of posteriors or either 0 (Column "Exact"), 2 p.p. (Column "Within 2 p.p.") or 5 p.p. (Column "Within 5 p.p."). Note that in Column "Within 5 p.p.," 0.33% of reported guesses are classified both as Bayesian and one-stage. We pool the data from both repeated model pairs.

Column "Exact" reports the shares of the reported vectors of posteriors that correspond exactly to those predicted by each updating rule. This analysis is considerably more stringent than the one in Section 4.2 as it requires a match with the point prediction for both signal realizations. To relax this demanding requirement, we also look at the share of observations for which the Euclidean distance between the reported vector of posteriors and the prediction vector is within 2 or 5 percentage points (Column "Within 2 p.p." and Column "Within 5 p.p."). These classifications capture between 42% and 59% of the reported vectors of posteriors.<sup>20</sup>

Despite using substantially more demanding criteria for classification, the data pattern remains closely aligned with the findings we report in our analysis of individual guesses. When allowing for a distance of at most 2 percentage points between the reported vector and the prediction, almost 40% of all reported vectors of posteriors are consistent with model selection for both signal realizations ("Best-fitting," "Worst-fitting," "Model 1," or "Model 2" in Figure 4 and Table 4). Additionally, the two most common predictions continue to be selecting the best-fitting model (23.33%) and one-stage updating (9.67%), while the vectors of posteriors consistent with Bayesian updating are rather rare (3.33%). Also, a few participants consistently report the prior over the state (2%).<sup>21</sup> We do not find evidence of any other frequent and consistent patterns in the data.

Analyzing the reported vectors of posteriors is also valuable in better distinguishing the different criteria for model selection. While selecting the best-fitting model for both signals is common, supporting model selection via maximum likelihood, consistently se-

 $<sup>^{20}</sup>$ Appendix Tables A3 and A4 replicate this classification separately for each model pair and show that the distributions are closely aligned. These classifications capture between 41% and 60% of the reported vectors for Model Pair 2, and between 43% and 60% of the reported vectors for Model Pair 3.

 $<sup>^{21}</sup>$ We do not include reporting the prior over the state as a category in Table 2 because it overlaps with other predictions for some model pairs. Overall, 6% of the reported guesses correspond to the prior, but only 2.8% for model pairs where there is no overlap with other updating rules.

lecting the worst-fitting model is rare. This suggests that such behavior is more likely a mistake in applying a model selection criterion rather than a systematic updating rule. Dogmatic model selection would imply that the same model is selected for both signals. We only find a few instances in which participants select the same model twice ("Model 1" or "Model 2" in Figure 4 and Table 4), indicating that the dogmatic criterion does not play a major role. Also, the frequencies with which each of the two models is selected are similar, indicating no systematic preference for one of the two models. The informative-ness criterion predicts that participants should select Model 2 for both signals in Model Pair 2 and Model 1 for both signals in Model Pair 3. Only 4.05% and 3.95% of vectors of posteriors are consistent with these predictions, respectively, suggesting this is not a predominant model selection criterion.

Selecting the same model for both signals could also be consistent with a stochastic version of model selection via maximum likelihood. We explore this possibility in Appendix C. First, to investigate the stochasticity in model selection, we examine the repeated tasks where participants observe the same signal. Among participants who select a model in both updating tasks (N=118), 16.1% select two different models. These inconsistencies imply that, on average, participants make a mistake in applying their model selection criterion with a probability of 8.83%. Next, we use this estimate and the reported vectors of posteriors to determine the shares of participants following the different model selection criteria but make mistakes with the estimated probability. Among participants that select models across signal realizations, we estimate that 73.10% follow the maximum likelihood criterion, 21.02% use either the dogmatic or informativeness criteria, and the residual 5.88% aim to consistently select the worst-fitting model. Therefore, this simple calibration exercise indicates that model selection in our data can be well described by a stochastic version of model selection rules.

Importantly, analyzing the reported vectors of posteriors also allows us to test for the presence of frequent Bayesian inconsistencies, meaning that posteriors reported across both signal realizations are consistently higher or lower than the prior. Such inconsistencies are a rather extreme deviation from Bayesian updating and are a crucial consequence of biases in model weighting that systematically assign disproportionate weight to the best-fitting or worst-fitting model. Figure 4 shows that a substantial share of the reported vectors of posteriors are Bayes-inconsistent, as they lie in the gray areas. When pooling data from both model pairs, we find that 50% of vectors are Bayes-inconsistent, of which 41% are in the direction of the best-fitting model and 9% in the direction of the worst-fitting model.<sup>22</sup> Hence, we find compelling evidence for the presence of Bayesinconsistencies, mostly driven by maximum likelihood selection. As a benchmark, note that another study where participants encounter only one model (Aina et al., 2024) hence, there are no reasons to expect frequent Bayes-inconsistencies—report only 6%

 $<sup>^{22}</sup>$ In Model Pair 2 and in Model Pair 3, respectively, 53% and 46% of the reported vectors are Bayesinconsistent. This difference is not statistically different (p-value = 0.206).

Bayes-inconsistent vectors of posteriors. This suggests that the presence of competing models, combined with systematic biases that distort model weights towards either the best- or worst-fitting model, are responsible for the large share of inconsistent posteriors across signals.

### 4.4 Consistency in the Use of Updating Rules

In the previous sections, we observed substantial heterogeneity in the reported guesses; however, a few rules can describe the majority of the reported posteriors: selecting one model, as well as the Bayesian and one-stage updating. In this section, we investigate whether participants consistently use any of these rules across tasks.

We have already presented some evidence of consistent application of updating rules in our analysis of vectors of posterior beliefs.<sup>23</sup> In this section, we extend our analysis to consider all seven updating tasks each participant completes. Table 5 shows how often participants use specific rules, allowing for a distance of at most 2 percentage points between the prediction and the reported guess. We also consider the possibility of reporting the prior over the state, as we see some evidence for such guesses in Figure 4. The table indicates that a large share of participants consistently adhere to one of these updating rules. When requiring participants to use the same rule for all seven guesses, we classify 40.34% of our sample. By allowing participants to deviate from the rule in one or two guesses, we classify 49% and 54% of participants, respectively. We can classify even 60% of participants when requiring a majority of guesses to follow one updating rule. Hence, we conclude that many participants consistently use a single updating rule.

In the appendix, we report a series of robustness tests from which we draw similar conclusions. Appendix Table A6 requires an exact match between the rule and the guess (instead of allowing for a distance of at most 2 percentage points) and Table A7 uses alternative approaches to classify participants based on average or median distances between their guesses and the predictions of the different updating rules. Moreover, we do not find that the updating rules participants use systematically change with experience.<sup>24</sup>

The observed pattern in Table 5 aligns with findings from the previous sections. While some participants put positive weight on both models by consistently following Bayesian and one-stage updating across tasks, the most common updating rule that is consistently applied corresponds to model selection, with 36% of participants using this rule in at least 6 out of 7 updating tasks. We then investigate the criteria these participants use for model selection. Figure 5 illustrates how frequently participants who select a model for all 7 tasks

 $<sup>^{23}</sup>$ Relatedly, we can also investigate the consistency of the guesses from the repeated tasks where the same signal is observed, as in Section 4.1. Appendix Table A5 reports this analysis and documents substantial consistency. Moreover, these estimates are in line with both the shares of guesses and vectors of posteriors reported in the previous two sections. For example, among these pairs of guesses, 28.7% corresponds to selecting the best-fitting model and 11.7% to one-stage updating.

<sup>&</sup>lt;sup>24</sup>Appendix Table A8 reports the joint distribution of used updating rules when comparing guessing the first and the last (before the repeated model pairs) tasks. This table reveals no systematic patterns, indicating no systematic change over time in the updating rules participants use.

Nr. Consistent Observations	Bayesian	One-stage	Model Selection	Prior
0	67.67	56.33	33.00	49.00
1	21.00	18.33	13.00	39.00
2	7.00	8.67	6.00	8.67
3	1.67	4.67	4.00	1.67
4	0.33	1.33	4.33	0.33
5	0.33	0.67	3.67	0.33
6	0.00	2.33	6.00	0.33
7	2.00	7.67	30.00	0.67
Total	100.00	100.00	100.00	100.00

Table 5: Consistency of Updating Rules

Notes. The table reports how often participants use specific updating rules. Since participants complete seven updating tasks, they can apply each rule between 0 and 7 times (Column "Nr. Consistent Observations"). We allow for a distance of at most 2 p.p. between the prediction and the reported guess. The columns display the distribution of frequencies for Bayesian updating, one-stage updating, model selection, and reporting the prior. For example, the column "Bayesian" shows the share of participants who report guesses that correspond to Bayesian updating in 0, 1, 2, 3, 4, 5, 6, and 7 tasks. Note that in some cases, the predictions for these updating rules overlap: Bayesian and one-stage overlap in 2.33% of all observations, prior and one-stage in 1.86% of all observations, and prior and model selection in 3.05% of all observations. In such cases, the observation counts for both rules. No participant can have a majority of consistent guesses for multiple updating rules.





*Notes.* The figure shows, among the 90 participants who selected a model in all 7 tasks (see Table 5), how frequently they selected the best-fitting model rather than the worst-fitting one.

choose the best-fitting model. The figure reveals that a majority of these participants (51.11%) select the best-fitting model in at least 6 out of 7 tasks, while only 17.78% select the worst-fitting model the majority of the time. No participant selects the worst-fitting model in all the tasks. Appendix Figure A4 supports this finding by conducting the same analysis for participants who selected a model in 6 out of 7 updating tasks. This systematic tendency to select the best-fitting model offers further support for the

maximum likelihood criterion over the dogmatic and informativeness criteria. Appendix Figure A5 reports how frequently participants selected the most informative model and reveals that only a few participants consistently follow this criterion. Taken together, these findings reinforce the interpretation, discussed in the previous section, that model selection via maximum likelihood generally describes well which model participants pick, though errors in evaluating model fit can sometimes lead to the selection of the worstfitting model.

### 4.5 Other Updating Rules

So far, we have focused on three main updating rules: Bayesian updating, one-stage updating, and model selection. These rules collectively explain most of our data: 72% of reported guesses align with these rules, allowing for a margin of error of at most 2 percentage points (Table 2). Moreover, participants display consistency in their use of these rules, with 53% applying the same rule for at least 5 out of 7 guesses (Table 5). Given that these rules account for a large share of the reported guesses, there is little room to explore the role of additional rules. Nonetheless, a small group of participants may consistently use alternative updating rules. In this section, we summarize our findings on these alternative updating rules and provide further details in Appendix D.

Less Extreme Forms of Over- and Underinference Model selection via maximum likelihood and one-stage updating reflect extreme over- and underinference about the models, respectively. Some participants might instead engage in less extreme versions of such biased inference about the models. In Appendix D.1, we present a theoretical framework capturing such rules and apply it to our data. This analysis reveals that some participants consistently use less extreme forms of over- or underinference but occasionally make errors in identifying the best-fitting model. Incorporating these updating rules enables us to classify an additional 16% of participants when requiring consistency across at least 5 out of 7 guesses, increasing the total share of classified participants to 70%.

**Signal-independent Model Weights** One-stage updating and dogmatic model selection assign model weights that do not depend on the signal. In Appendix D.2, we study the broader class of updating rules sharing this property. However, we find no evidence that any other signal-independent updating rules are consistently applied in our data.

Aggregating Competing Models into a Single Model Instead of aggregating the predictions of the different models, individuals could first combine the competing models into a single compound model and then use this model to update their beliefs based on the observed signal. For a Bayesian, these two approaches are equivalent if the compound model is derived using the prior over models. However, biases in updating with such a compound model result in deviations from the Bayesian benchmark that differ fundamentally from those arising from the biases in model weighting discussed in Section

2. Most importantly, updating based on a compound model predicts posteriors to be Bayes-consistent, even if the updating process or the compounding procedure is biased. However, we find that half of the reported vectors of posteriors are Bayes-inconsistent. Appendix D.3 provides further evidence against the consistent application of such rules that are based on a compound model.

**Use of Multiple Rules** Participants might alternate between different updating rules, such as using one-stage updating in some instances and Bayesian updating in others. However, as shown in Appendix D.4, participants rarely use multiple updating rules, and no systematic patterns emerge in their combinations.

Hence, in addition to one-stage updating, Bayesian updating, and model selection, we find that some participants employ less extreme forms of over- or underinference about the models. However, our analysis finds no evidence that any other rules are applied frequently or consistently in our data.

## 4.6 Determinants of Model Selection versus Model Weighting

Having established that many participants consistently use specific updating rules, we now investigate whether different groups of participants employ different rules. Our main focus is on understanding the characteristics of individuals who engage in model selection rather than consistently assigning positive weight to both models, as these individuals are particularly vulnerable to persuasion using models.<sup>25</sup>

Table 6 presents the results of regressing a dummy variable for model selection on various participant characteristics. Our analysis focuses on the 162 participants who consistently use one of the updating rules in at least 5 out of 7 tasks. Appendix Tables A9, A10, A11, and A12 demonstrate that these results are robust when focusing on participants who used one rule in 4, 6, or 7 tasks, or when considering the entire sample and counting the number of times a participant selected a model.

We begin our analysis by considering the main demographic characteristics of our participants. Specifications (1) to (3) provide coefficient estimates for gender, age, and education. We find that women are relatively more likely to select models rather than weight them. Next, we examine self-reported political attitudes. Model selection has been theoretically linked to holding particularly extreme views. Hence, since the relationship between model selection and political attitudes could be nonlinear—where individuals with more extreme positions are more likely to select models—we include dummies for extreme political positions, "Very Liberal" and "Very Conservative". Specifications (5) and (6) confirm that being very conservative is associated with more frequent model selection, while no statistically significant association is found for strongly liberal views.

<sup>&</sup>lt;sup>25</sup>Appendix Figure A6 presents the characteristics of participants who apply different updating rules, also distinguishing between the two main rules that put positive weight on both models—Bayesian updating and one-stage updating—and model selection.

Dependent Variable				M	odel Se	election			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Female	0.180***								0.147**
	(0.069)								(0.069)
Age		-0.005							-0.005*
		(0.003)							(0.003)
Education			-0.063						-0.009
			(0.047)						(0.042)
Very Liberal				-0.050					-0.067
0				(0.077)					(0.071)
Very Conservative					0.205**	c .			0.091
0					(0.083)	)			(0.088)
Cognitive Reflection Test						-0.132***			-0.109***
						(0.025)			(0.026)
REI Rationality Scale							-0 161***	:	-0 107*
							(0.056)		(0.056)
BEL Experientiality Scale								0.054	0.002
Tell Experientianty Seale								(0.045)	(0.041)
Observations	162	162	162	162	162	162	162	162	162
$R^2$	0.042	0.015	0.012	0.003	0.016	0.114	0.041	0.011	0.185

#### Table 6: Explaining Model Selection

Notes. The table shows coefficient estimates from linear regressions of an indicator of model selection on different explanatory variables. The sample consists of the 162 participants who consistently used one updating rule (Bayesian, one-stage, model selection, prior) in at least 5 out of 7 tasks, allowing for a distance of at most 2 p.p between the prediction and the reported guess (see Table 5). "Female," "Very Liberal," and "Very Conservative" are indicator variables, "Age" is in years (19 and 70), "Education" is from 1 = "High school" to 4 = "PhD or higher," "Cognitive Reflection Test" are scores from 0 to 3, and Rational Experiental Inventory (REI) rationality and experientiality scales are from 1 to 5. Heteroskedasticity-robust standard errors are shown in parentheses. \* p < 0.10, \*\* p < 0.05, \*\*\* p < 0.01

Biases in belief updating have also been linked to cognitive abilities (Oechssler et al., 2009; Hoppe and Kusterer, 2011; Augenblick et al., 2021; Enke and Graeber, 2023; Oprea, 2025). We collect different psychological assessments of cognitive ability and thinking styles at the end of our study. First, we consider the role of the Cognitive Reflective Test (CRT), which measures the tendency to override incorrect intuitive responses (Frederick, 2005). Specification (6) shows that participants with high CRT scores are less likely to select models. This can be explained by noting that individuals with higher CRT scores tend to engage in more deliberative and analytical processing of information, making them more likely to weight competing models rather than selecting a single one. Second, we consider the Rational-Experiential Inventory (REI), which assesses participants' thinking styles (Keaton, 2017). This scale measures the extent to which participants engage in two modes of thinking: intuitive thinking (Experientiality Scale) and logical thinking (Rationality Scale). Specifications (7) and (8) indicate that participants with higher scores on the REI Rationality Scale are less likely to select models, while there is no significant effect for the Experientiality Scale. This further suggests that individuals who favor a methodical and rational approach to processing information are more inclined to weight multiple models rather than selecting a single one.

Finally, in Specification (9), we assess all these factors in a single regression. We find that CRT scores and REI remain significant predictors when all factors are included. However, the association with being very conservative is no longer statistically significant when controlling for all these measures. Hence, differences in updating based on political position seem to be shaped by underlying differences in thinking style.

These findings suggest that cognitive thinking styles, as measured by the CRT and REI, might be particularly relevant in determining whether a participant chooses to weight multiple models or select a single one, and, hence, how vulnerable to persuasion using models this individual can be. As discussed in Appendix E, these updating rules involve a trade-off between precision and response time. Participants who are classified as consistently engaging in model selection or one-stage updating report beliefs that deviate from the Bayesian benchmark, but also exhibit substantially shorter response times. Specifically, the median response time for Bayesian updating is more than three times higher than that for model selection. It is possible that the participants' cognitive traits influence their approach to this trade-off between speed and accuracy in belief updating.

## 4.7 Evidence from Other Updating Environments

Processing new information when confronted with competing models could occur in different environments. So far, we have focused on the case with an objective prior distribution over the models and readily available model predictions. We took this as a starting point because it mirrors realistic features of many real-world settings and it allows us to reliably measure biases in model weighting. The results presented thus far established the relevance of model selection via maximum likelihood. In this section, we assess the generalizability of our findings by independently varying these two design features.

First, we examine a setting where participants are not provided with an exogenous prior distribution over models. This also aligns with the literature on ambiguous beliefs, which does not specify model priors and assumes model selection via maximum likelihood. In one treatment variation, the *No-Prior* condition, we study whether model selection remains a prevalent updating rule in contexts where participants are not provided with an exogenous prior over the models. Note that while model selection remains well-defined in this treatment condition, identifying Bayesian and one-stage updating requires assumptions about subjective model priors; we pre-registered that we would focus on Bayesian and one-stage updating with uniform prior over models.

Second, we examine a setting where model predictions are not immediately available, requiring participants to exert effort to access each of them. While providing model pre-

dictions reflects many real-world settings where individuals encounter a model alongside its prediction, in other cases, such predictions may not always be readily available and involve cognitive, attention, or search costs to access them. To test whether our findings extend to such settings, we introduce a second treatment variation, the *Click* condition, where participants can obtain each prediction by clicking a corresponding button five times in a row ("R" button for the first model and "B" button for the second model). This approach introduces real effort costs in a highly controlled manner (e.g., Ariely et al., 2009; DellaVigna and Pope, 2018), capturing any potential costs associated with accessing model predictions. This treatment condition allows us to examine whether participants actively seek out model predictions at a small cost and whether model selection remains prevalent in settings where model predictions are less salient and require deliberate effort to access them. However, note that this design limits our ability to identify updating rules unless most participants reveal the predictions.<sup>26</sup>

In addition to the *No-Prior* and the *Click* conditions, we replicate our initial study. As the data in all three conditions were collected simultaneously, the *Baseline* condition provides a benchmark for using different updating rules compared to the novel conditions. This second study closely replicates the design described in Section 3. We limit the number of guesses to two, thereby shortening the completion time. We use Model Pair 2 and Model Pair 3 described in Table 1 and Figure 4. This pre-registered experiment was conducted on Prolific in February 2025 with 592 participants across all treatments; Appendix F presents details about the experimental design and provides additional results.<sup>27</sup>

We begin our analysis by looking at the demand for model predictions. Table 7 presents the frequencies with which participants revealed zero, one, or both model predictions in the *Click* treatment. Only in 6% of all updating tasks, participants proceed without acquiring any predictions, while in 10% of tasks, participants reveal the prediction of one model and in 84% of tasks, they reveal predictions for both models. Interestingly, when participants reveal only one model prediction, it is more often the prediction of the best-fitting model (6.40% vs. 3.20%; t = 2.12, p = 0.046). We conclude that participants generally are willing to take a deliberate, costly action to access model predictions. The high rate of model prediction acquisition also ensures the comparability between the *Click* and *Baseline* conditions.

Next, we study whether the updating patterns documented in our initial study extend to these alternative updating environments. Table 8 reports the classification of guesses consistent with Bayesian updating, one-stage updating, and selecting either the best- or worst-fitting model. As specified in our pre-registration, we report the shares of guesses consistent with these updating rules allowing for a small deviation of at most 2 percentage

 $<sup>^{26}</sup>$ We cannot determine whether participants who do not reveal any model predictions are following one of the discussed updating rules or suffer from biases in deriving model predictions instead.

<sup>&</sup>lt;sup>27</sup>The pre-registration plan is available at https://aspredicted.org/wqtd-68zm.pdf.

Nr. Predictions Revealed	%	95%-CI
Zero	6.10	[2.73, 9.48]
One, Total	9.59	[5.58, 13.61]
One, Best-fitting	6.40	[3.39, 9.40]
One, Worst-fitting	3.20	[1.18, 5.22]
Both	84.01	[78.95, 89.08]

Table	$7 \cdot$	Demand	for	Model	Predictions
ranc		Domana	TOT	mouti	I I CUICUOID

*Notes* This table reports the frequencies with which participants revealed zero, one, or both model predictions in the *Click* condition (N=344). For revealing one prediction, the table gives the overall frequency (row "One, Total"), as well as the frequency in which only the predictions of the best-fitting and only the worst-fitting models are revealed (rows "One, Best-fitting" and "One, Worst-fitting"). Columns "%" and "95%-CI" report the shares and corresponding 95% confidence intervals, using standard errors clustered at the individual level. We pool the data from both updating tasks.

Type of Guess	Baseline		I	No-Prior	Click		
	%	95%-CI	%	95%-CI	%	95%-CI	
Bayesian	3.73	[1.90, 5.56]	4.57	[2.44, 6.69]	4.94	[2.55, 7.34]	
One-stage	12.19	[8.10,  16.28]	13.47	[9.46, 17.48]	15.99	[11.19, 20.78]	
Best-fitting	44.03	[38.02, 50.04]	43.61	[38.04, 49.18]	44.48	[37.79, 51.16]	
Worst-fitting	11.44	[7.98, 14.90]	10.05	[7.01,  13.08]	11.34	[7.51,  15.16]	
Total	71.39		71.70		76.75		

Table 8: Classification of Guesses by Treatment (Second Study)

*Notes.* The table reports the shares of guesses that fall within 2 p.p. around each point prediction (Bayesian, one-stage, best-fitting, and worst-fitting) for each treatment (*Baseline, No-Prior, Click*). Columns "%" and "95%-CI" report the shares and corresponding 95% confidence intervals, using standard errors clustered at the individual level. We pool the data from both updating tasks.

points, pooling together data from both updating tasks.<sup>28</sup> We find model selection to be prevalent for all treatments; 55% to 57% of reported guesses correspond to selecting only one model. In particular, selecting the best-fitting model is the most frequent updating rule, with a frequency of 44% in all treatments, followed by one-stage updating with a frequency ranging from 12% to 16%.<sup>29</sup>

Our analysis reveals no systematic differences between the treatment variations. None of the shares of guesses from either *No-Prior* or *Click* conditions are statistically significantly or meaningfully different from those of *Baseline* (Appendix Tables F5 and F6).

 $<sup>^{28}</sup>$ Appendix Table F1 reports these shares requiring exact matching between guesses and predictions, while Appendix Tables F3 and F4 replicate Table 8 for each model pair. We do observe any meaningful differences. Importantly, data from the *Baseline* condition closely replicates our findings in the main study; as a benchmark, Appendix Table F2 reports the shares of the updating rules for Model Pair 2 and Model Pair 3 in the initial study.

 $<sup>^{29}</sup>$ In the *No-Prior* condition, participants could use Bayesian or one-stage updating based on subjective model priors other than the uniform prior. However, two findings suggest little room for such updating. First, we find no evidence of participants frequently assigning weights to models beyond those discussed in Table 8; indeed, we do not observe other frequent updating patterns that could be attributed to other frequent model priors (Appendix Figures F1 and F2). Moreover, when we consider Bayesian and one-stage updating assuming equal prior over models, we find that the prevalence of these updating rules in the *No-Prior* condition matches their prevalence in the *Baseline* condition (Table 8).

Furthermore, this is also confirmed by looking at the distributions of model weights,  $\rho$  (Appendix Figure F1). Taken together, our results suggest that the findings of our initial study extend to updating environments without a given objective prior over the models, as well as to situations where model predictions are not immediately available but require deliberate actions to access them.

Given the consistency across treatments, we pool the data to examine broader updating patterns. This confirms our previous findings. First, participants apply updating rules consistently across tasks (Appendix Table F7). Second, model selection is primarily driven by choosing the best-fitting model (Appendix Tables F8 and F9, replicating the analysis of Table 6).

# 5 Discussion and Conclusion

This paper studies how individuals update their beliefs when confronted with competing data-generating processes, or models, that could explain the data and provide conflicting predictions. We design and implement an experimental study to identify the weights individuals assign to different models when updating their beliefs in such situations.

We offer three key insights. First, we provide strong evidence supporting the most common assumption in the literature on model persuasion: model selection via maximum likelihood. This approach assumes that individuals place full weight on a single model, specifically, the one that best fits the data. Our findings show that model selection via maximum likelihood is the most frequently applied updating rule across different updating environments, including those with and without a pre-specified model prior. Moreover, participants apply this rule consistently across updating tasks. These results not only validate the assumption of theoretical papers about model persuasion but also hold important implications for policy-oriented research. In particular, understanding how people aggregate the predictions from competing models can shed light on the cognitive processes that drive polarization. While assigning positive weights to multiple models might promote more nuanced perspectives, the tendency to select only one model could contribute to the rise of extreme beliefs and polarized views (Izzo et al., 2023; Aina, 2024; Schwartzstein and Sunderam, 2024). This aligns with our findings that participants who describe themselves as very conservative are more likely to engage in model selection.

Second, while model selection via maximum likelihood describes well the belief updating of many participants, our study uncovers additional belief-updating patterns that inform future research, especially in developing more descriptively accurate models to assess policy impact and welfare considerations. On one hand, we observe that individuals often make errors when applying model selection criteria, at times selecting the worst-fitting model instead of the best-fitting one. These mistakes differ from the usual approach to modeling errors, which involves frequent but minor deviations from the predictions of the underlying updating rule. In contrast, errors in model selection are of large magnitude. Considering the examples discussed in the introduction, such errors could lead to fundamentally different conclusions about future asset returns, the legitimate winner of an election, or vaccine safety. On the other hand, we find that some participants consistently apply updating rules other than selecting the best-fitting model. In particular, one-stage updating emerges as the second most frequently applied rule in our study, and we believe it deserves attention for future research. From the perspective of over- and underinference about the models, one-stage updating stands as the opposite extreme of model selection via maximum likelihood: While model selection via maximum likelihood represents an extreme form of overinference about the models, one-stage updating represents an extreme form of underinference. A direct implication of errors in applying model selection criteria and one-stage updating is that the distribution of beliefs about the state of the world is multimodal, with peaks at the predictions of the best-fitting model, the worst-fitting model, and the one-stage updating. Consequently, average beliefs fail to adequately represent the population's belief distributions, as in Bordalo et al. (2023).

Third, our data shows the systematic emergence of Bayes-inconsistencies: when confronted with conflicting models, individuals become more confident about a certain state of the world regardless of the observed signal. Bayes-inconsistencies are a central prediction of certain biases in model weighting, particularly those that lead to strong overinference about the models. This finding demonstrates the potential impact of persuasion using models. Bayes-inconsistencies contrast with the constraint the persuader faces in classic models of persuasion, such as Bayesian persuasion. It shows how it is possible to persuade individuals to adopt Bayes-inconsistent beliefs by presenting them with conflicting models, as proposed by Aina (2024). For instance, a politician might convince voters that she won an election regardless of the reported outcome by introducing conflicting narratives about the election system. Voters could selectively adopt these narratives depending on the result, concluding that the system is fair if she wins but rigged if she loses. Analogously, a financial advisor could convince investors to always invest in certain assets by providing conflicting ways of interpreting past financial data.

The observation of frequent Bayes-inconsistencies also highlights that biases in model weighting can have fundamentally different implications than those associated with entertaining a single model. There are no theoretical reasons to expect this type of inconsistent beliefs when individuals update their beliefs based on a single model, even when exhibiting over- or underreaction to the new information. These observations are also important for researchers studying belief updating, as it is often challenging to determine whether individuals consider multiple models and, if so, which models they consider. We provide two valuable insights on this perspective. First, studying biases in belief updating in a single-model framework, when individuals may actually entertain multiple models, can lead to inaccurate conclusions. For example, frequent Bayesian inconsistencies would be attributed to belief updates moving in the wrong direction and, consequently, to low data quality. Therefore, it is important to carefully consider the possibility that subjects entertain multiple models when analyzing belief data. Second, even when models are not

directly observable, one can assess whether subjects entertain multiple models by testing for frequent occurrences of Bayesian inconsistencies. Such inconsistencies serve as a clear indicator that participants are considering multiple models but exhibit biases in how they weight them.

Several compelling questions remain to be addressed. For instance, we deliberately focus on a setting where individuals have no personal stake in the outcomes. Introducing stakes could lead to additional motives; for example, individuals might be inclined to select models that offer the most optimistic predictions regarding their potential rewards. Moreover, we limit our analysis to a context in which individuals update their beliefs in response to only one signal. This approach leaves the dynamic aspects of this process unexplored, raising questions about how model selection might evolve in response to multiple signals over time: would individuals re-evaluate models based on cumulative signals, assign more importance to the more recent signal, or adhere to the first model selected? This paper establishes a foundation to understand belief updating in these settings, paving the way for further research about decision-making when more than one model informs our interpretation of what we observe.

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# Supplemental Appendix for "Weighting Competing Models"

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## A Appendix: Additional Results



Figure A1: Cumulative Distribution of Within-Task Consistency Notes. Cumulative distribution of the measure of consistency  $\Delta$  Posterior, defined as the absolute difference between the reported posteriors given the same signal for the same model pair.



#### (a) Conditional on Observing a Purple Ball



(b) Conditional on Observing an Orange Ball

Figure A2: Comparison of Posteriors across Model Pairs

*Notes.* The figures represent the model predictions and average reported guesses for Model Pair 3 and Model Pair 5 and each signal realization. The prior over the state are represented by a small black circle; the model predictions of Model 1 are represented by a red circle, while the ones of Model 2 are represented by a blue square. The average reported posterior for each case is described by a black diamond.



Figure A3: Mean Guesses and Bayesian Predictions

*Notes.* The figure plots the mean reported guess that bag A is selected and the Bayesian prediction for model pair and signal realization. While the difference between mean belief and the Bayesian prediction is often small, it is statistically significantly different from zero for 8 out of the 10 cases.



Figure A4: Errors in Model Selection: Robustness

*Notes.* The figure shows, among the 18 participants who selected a model in 6 out of 7 tasks (see Table 5), how frequently they selected the best-fitting model rather than the worst-fitting one.



Figure A5: Model Selection based on Informativeness

*Notes.* The figure shows, among the 90 participants who selected a model in all 7 tasks (see Table 5), how frequently they selected the most informative model.



Figure A6: Individual Characteristics by Updating Rules

Notes. The figures show the average individual characteristics of participants who consistently use different updating rules. The sample consists of the 158 participants who consistently used the Bayesian updating, one-stage updating, or model selection in at least 5 out of 7 tasks, allowing for a distance of at most 2 p.p between the prediction and the reported guess (see Table 5). Due to the small number of observations, we do not include participants who consistently reported the prior. "Female", "Very Liberal" and "Very Conservative" are indicator variables, "Age" is in years (19 and 70), "Education" is from 1 = "High school" to 4 = "PhD or higher", "Cognitive Reflection Test" are scores from 0 to 3, and the Rational Experiental Inventory (REI) rationality and experientiality scales range from 1 to 5.

Type of Guess	% Exact	Within 2 p.p.
Bayesian	3.15	5.90
One-stage	16.51	23.20
Best-fitting	42.86	47.84
Worst-fitting	12.32	13.37
Within	17.56	
Outside	7.60	
Total	100.00	88.21

Table A1: Classification of Guesses: Consistent Guesses

*Notes.* Consistent participants are classified as reporting a guess within 2 p.p. for the same task as described in Section 4.1. Note that 1.7% of reported guesses are classified both as Bayesian and as one-stage updating, not included in the total.

Dependent Variable		Sel	ects Mode	l 1	
	(1)	(2)	(3)	(4)	(5)
Best-fitting	$0.454^{***}$		$0.445^{***}$		0.447***
	(0.038)		(0.040)		(0.040)
Magt I. fam		0 105***	0.029		0.006
Most informative		-0.105	-0.032		0.006
		(0.036)	(0.033)		(0.078)
Model Pair 2				0.130***	0.098**
				(0.045)	(0.039)
				(0.010)	(0.000)
Model Pair 3				-0.036	0.007
				(0.055)	(0.090)
				,	
Model Pair 4				0.037	-0.011
				(0.064)	(0.063)
Model Pair 5				-0.056	0.002
				(0.060)	(0.076)
Constant	0 252***	0 538***	0 268***	0 453***	0 224***
Constant	(0.024)	(0.021)	(0.021)	(0.433)	(0.0224)
	(0.024)	(0.021)	(0.031)	(0.043)	$\frac{(0.030)}{050}$
Observations	956	956	956	956	956
$R^2$	0.206	0.025	0.207	0.021	0.214

Table A2: Criteria for Model Selection (Within 2 p.p.)

Notes. The table shows coefficient estimates from linear regressions of an indicator of selecting Model 1 on different explanatory variables. The sample consists of the 956 guesses that correspond to model selection allowing for a distance of at most 2 p.p. between the prediction and the reported guess (see Table 2). "Best-fitting" is an indicator for Model 1 having a higher fit than Model 2 given the observed signal. "Most informative" is an indicator for Model 1 being more informative about the state than Model 2 given the observed signal. "Model Pair 2" to "Model Pair 5" are model pair fixed effects that capture any factors that are unconditional on the signal. Standard errors are clustered on the individual level (179 clusters) and are shown in parentheses. \* p < 0.10, \*\* p < 0.05, \*\*\* p < 0.01

Type of Vector		Exact V		ithin 2 p.p.	W	ithin 5 p.p.
	%	95%-CI	%	95%-CI	%	95%-CI
Bayesian	3.38	[0.43, 6.32]	4.73	[1.27, 8.19]	7.43	[3.16, 11.71]
One-stage	4.73	[1.27, 8.19]	7.43	[3.16, 11.71]	10.81	[5.75, 15.87]
Best-fitting	19.59	[13.12, 26.06]	25.00	[17.94, 32.06]	26.35	[19.17, 33.53]
Worst-fitting	1.35	[0.00, 3.23]	2.03	[0.00, 4.32]	2.03	[0.00, 4.32]
Model 1	4.73	[1.27, 8.19]	6.76	[2.67, 10.85]	6.76	[2.67, 10.85]
Model 2	4.05	[0.84, 7.27]	4.05	[0.84, 7.27]	4.05	[0.84, 7.27]
Prior	2.70	[0.06, 5.35]	2.70	[0.06, 5.35]	2.70	[0.06, 5.35]
Total	40.53		52.70		60.13	

Table A3: Classification of Vectors of Posteriors (Model Pair 2)

*Notes.* The table reports the shares of vectors of posteriors that have an Euclidean distance from the prediction vector of posteriors or either 0 (Column "Exact"), 2 p.p. (Column "Within 2 p.p.") or 5 p.p. (Column "Within 5 p.p."). We only include the data from Model Pair 2.

Type of Vector	Exact		W	ithin 2 p.p.	Within 5 p.p.		
	%	95%-CI	%	95%-CI	%	95%-CI	
Bayesian	1.97	[0.00, 4.21]	1.97	[0.00, 4.21]	3.29	[0.42, 6.16]	
One-stage	7.24	[3.07, 11.40]	11.84	[6.65, 17.04]	14.47	[8.82, 20.13]	
Best-fitting	19.74	[13.34, 26.14]	21.71	[15.08, 28.34]	23.03	[16.26, 29.80]	
Worst-fitting	3.29	[0.42, 6.16]	3.29	[0.42, 6.16]	3.29	[0.42, 6.16]	
Model 1	3.95	[0.82, 7.08]	3.95	[0.82, 7.08]	4.61	[1.24, 7.98]	
Model 2	5.26	[1.67, 8.85]	7.89	[3.56, 12.23]	7.89	[3.56, 12.23]	
Prior	1.32	[0.00, 3.15]	1.32	[0.00, 3.15]	1.32	[0.00, 3.15]	
Total	42.77		51.97		57.90		

Table A4: Classification of Vectors of Posteriors (Model Pair 3)

*Notes.* The table reports the shares of vectors of posteriors that have an Euclidean distance from the prediction vector of posteriors or either 0 (Column "Exact"), 2 p.p. (Column "Within 2 p.p.") or 5 p.p. (Column "Within 5 p.p."). Note that in Column "Within 5 p.p.", 0.33% of reported guesses are classified both as Bayesian and one-stage. We only include the data from Model Pair 3.

	Bayesian	One-stage	Best-fitting	Worst-fitting	Other
Bayesian	2.33	0.00	0.00	0.00	1.33
One-stage	1.33	11.67	0.00	0.00	2.67
Best-fitting	0.67	1.00	28.67	2.67	7.33
Worst-fitting	0.00	0.33	3.67	4.33	1.33
Other	2.67	3.33	3.00	1.00	20.67

Table A5: Classification of Guesses Repeated Model Pair, Same Signal (Within 2 p.p.)

*Notes* This table uses data from the repeated model pairs (Model Pair 2 and Model Pair 3) for cases where the individual observed the same signal. For each pair of guesses, we classify participants and allow for a distance of at most 2 p.p between the prediction and the reported guess.

Nr. Consistent Observations	Bayesian	One-stage	Model Selection	Prior
0	91.00	72.33	40.33	68.33
1	5.67	13.33	10.33	28.00
2	1.00	3.33	6.00	1.67
3	0.67	2.67	5.00	0.33
4	0.00	1.00	3.33	0.33
5	0.00	1.67	4.67	0.33
6	0.00	3.33	7.67	0.33
7	1.67	2.33	22.67	0.67
Total	100.00	100.00	100.00	100.00

Table A6: Consistency of Updating Rules (Exact)

*Notes.* The table reports how often different participants use specific updating rules. Since participants complete 7 updating tasks, they can apply each rule between 0 and 7 times (Column "Nr. Consistent Observations"). We require an exact match between the prediction and the reported guess. The columns display the distribution of frequencies for the Bayesian updating, one-stage updating, model selection, and reporting the prior. For example, the column "Bayesian" shows the share of participants who report guesses that correspond to the Bayesian rule in 0, 1, 2, 3, 4, 5, 6, and 7 tasks.

Updating Rule	Mean: Exact	Mean: 2 p.p.	Median: Exact	Median: 2 p.p.
Bayesian	1.67	2.00	1.67	2.67
One-stage	2.33	9.00	8.33	12.00
Model selection	22.67	33.67	38.33	44.00
Prior	0.67	0.67	1.67	1.67
Total	27.33	45.33	50.00	60.33

Table A7: Consistency of Updating Rules: Alternative Approaches

*Notes.* The table classifies participants according to the updating rules they use in the 7 updating tasks. First, we calculate the distance between the reported guess and each prediction for every observation. For each participant, we then calculate the average and median distance to each prediction. In the column "Mean: Exact," we classify participants if their mean distance to an updating rule is 0. In the column "Mean: 2 p.p.," we allow the average distance of at most 2 p.p.. "Median: Exact" and "Median: 2 p.p." employ a similar approach but use the median distance. Note that "Best- or worst-fitting" corresponds to selecting either the best- or worst-fitting model. The row "Total" gives the share of participants we can classify in these updating rules.

	Bayesian	One-stage	Best-fitting	Worst-fitting	Other	Total
Bayesian	2.67	0.00	0.00	0.00	3.33	6.00
One-stage	1.00	10.00	1.00	0.67	4.33	17.00
Best-fitting	0.67	3.33	19.33	6.67	5.00	35.00
Worst-fitting	0.00	0.67	6.67	4.67	2.33	14.33
Other	0.67	5.67	3.67	2.67	15.00	27.67
Total	5.00	19.67	30.67	14.67	30.00	100.00

Table A8: Classification of Guesses in Task 5 and Task 9 (Within 2 p.p.)

*Notes* This table uses data from the first updating task (rows) and the last updating tasks before repetitions (columns) with multiple models. For each pair, we classify participants and allow for a distance of at most 2 p.p. between the prediction and the reported guess.

Dependent Variable				М	odel Se	election			
-	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Female	$0.153^{**}$								$0.117^{*}$
	(0.066)								(0.066)
Age		-0.004							-0.004
		(0.003)							(0.003)
Education			-0.084*						-0.032
			(0.044)						(0.042)
Very Liberal				-0.022					-0.035
-				(0.073)					(0.068)
Very Conservative					0.209**				0.096
U U					(0.082)				(0.086)
Cognitive Reflection Test						-0.133***	¢		-0.115***
0						(0.024)			(0.026)
<b>REI</b> Rationality Scale							-0.140***	k	-0.085
U							(0.054)		(0.053)
REI Experientiality Scale	)							0.045	-0.005
r								(0.042)	(0.038)
Observations	181	181	181	181	181	181	181	181	181
$R^2$	0.030	0.010	0.021	0.001	0.015	0.116	0.031	0.007	0.167

#### Table A9: Explaining Model Selection: Robustness

Notes. The table shows coefficient estimates from linear regressions of an indicator of model selection on different explanatory variables. The sample consists of the 181 participants who consistently used one updating rule (Bayesian, one-stage, model selection, prior) in at least 4 out of 7 tasks, allowing for a distance of at most 2 p.p between the prediction and the reported guess (see Table 5). "Female," "Very Liberal," and "Very Conservative" are indicator variables, "Age" is in years (19 and 70), "Education" is from 1 = "High school" to 4 = "PhD or higher," "Cognitive Reflection Test" are scores from 0 to 3, and Rational Experiental Inventory (REI) rationality and experientiality scales are from 1 to 5. Heteroskedasticity-robust standard errors are shown in parentheses. \* p < 0.10, \*\* p < 0.05, \*\*\* p < 0.01

Dependent Variable				М	odel Se	election			
-	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Female	$0.168^{**}$								$0.144^{**}$
	(0.073)								(0.072)
Age		-0.006*							-0.006**
		(0.003)							(0.003)
Education			-0.075						-0.012
			(0.048)						(0.043)
Very Liberal				-0.031					-0.065
				(0.079)					(0.074)
Very Conservative					0.198**				0.098
u u u u u u u u u u u u u u u u u u u					(0.089)				(0.097)
Cognitive Reflection Test						-0.131***	¢		-0.109***
0						(0.026)			(0.028)
REI Rationality Scale							-0.173***	k	-0.120*
0							(0.061)		(0.062)
REI Experientiality Scale	•							0.051	-0.001
								(0.047)	) (0.043)
Observations	147	147	147	147	147	147	147	147	147
$R^2$	0.036	0.027	0.017	0.001	0.015	0.110	0.046	0.009	0.194

#### Table A10: Explaining Model Selection: Robustness

Notes. The table shows coefficient estimates from linear regressions of an indicator of model selection on different explanatory variables. The sample consists of the 147 participants who consistently used one updating rule (Bayesian, one-stage, model selection, prior) in at least 6 out of 7 tasks, allowing for a distance of at most 2 p.p between the prediction and the reported guess (see Table 5). "Female," "Very Liberal," and "Very Conservative" are indicator variables, "Age" is in years (19 and 70), "Education" is from 1 = "High school" to 4 = "PhD or higher," "Cognitive Reflection Test" are scores from 0 to 3, and Rational Experiental Inventory (REI) rationality and experientiality scales are from 1 to 5. Heteroskedasticity-robust standard errors are shown in parentheses. \* p < 0.10, \*\* p < 0.05, \*\*\* p < 0.01

Dependent Variable				M	odel Se	election			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Female	0.121								0.088
	(0.080)								(0.077)
Age		-0.009**							-0.008**
		(0.004)							(0.003)
Education			-0.089*						-0.013
			(0.051)						(0.043)
Very Liberal				-0.114					-0.109
U U				(0.089)					(0.079)
Very Conservative					0.182*				0.089
U U					(0.097)	)			(0.104)
Cognitive Reflection Test						-0.148***	¢		-0.114***
0						(0.027)			(0.029)
REI Rationality Scale							-0.196***	*	-0.160**
							(0.065)		(0.063)
REI Experientiality Scale	•							0.080	0.019
								(0.049)	(0.045)
Observations	121	121	121	121	121	121	121	121	121
$R^2$	0.019	0.059	0.026	0.015	0.014	0.140	0.063	0.025	0.257

Table A11: Explaining Model Selection: Robustness

Notes. The table shows coefficient estimates from linear regressions of an indicator of model selection on different explanatory variables. The sample consists of the 121 participants who consistently used one updating rule (Bayesian, one-stage, model selection, prior) in 7 out of 7 tasks, allowing for a distance of at most 2 p.p between the prediction and the reported guess (see Table 5). "Female," "Very Liberal," and "Very Conservative" are indicator variables, "Age" is in years (19 and 70), "Education" is from 1 ="High school" to 4 = "PhD or higher," "Cognitive Reflection Test" are scores from 0 to 3, and Rational Experiental Inventory (REI) rationality and experientiality scales are from 1 to 5. Heteroskedasticityrobust standard errors are shown in parentheses. \* p < 0.10, \*\* p < 0.05, \*\*\* p < 0.01

Dependent Variable				Nr. N	Model S	election			
-	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Female	$0.795^{**}$								0.669*
	(0.345)								(0.352)
Age		-0.024							-0.030*
		(0.015)							(0.015)
Education		-	-0.564***						-0.372*
			(0.217)						(0.219)
Very Liberal				0.146					0.026
-				(0.389)					(0.373)
Very Conservative					0.320				0.406
					(0.672)				(0.645)
Cognitive Reflection Test						-0.516***	¢		-0.462***
0						(0.152)			(0.156)
<b>REI</b> Rationality Scale							-0.605**	k	-0.361
U U							(0.289)		(0.291)
REI Experientiality Scale	:							-0.008	-0.179
I J LITT								(0.210)	(0.200)
Observations	300	300	300	300	300	300	300	300	300
$R^2$	0.017	0.009	0.021	0.000	0.001	0.035	0.013	0.000	0.080

#### Table A12: Explaining Model Selection: Robustness

Notes. The table shows coefficient estimates from linear regressions of the number of times a participant selected one model on different explanatory variables. The sample consists of all 300 participants. "Female," "Very Liberal," and "Very Conservative" are indicator variables, "Age" is in years (19 and 70), "Education" is from 1 = "High school" to 4 = "PhD or higher," "Cognitive Reflection Test" are scores from 0 to 3, and Rational Experiental Inventory (REI) rationality and experientiality scales are from 1 to 5. Heteroskedasticity-robust standard errors are shown in parentheses. \* p < 0.10, \*\* p < 0.05, \*\*\* p < 0.01

## **B** Appendix: Heterogeneity in Model Pairs

In this appendix, we explore heterogeneity in the use of updating rules across model pairs. Table 1 presents the model pairs participants encountered in the experiment and Figure B1 illustrates their point predictions. We examine whether characteristics of the models that differ across model pairs systematically influence updating. This analysis is exploratory because: i) there are no existing theories that predict how and which features, beyond fit and informativeness, should affect updating, ii) there is no clear and unified way to categorize differences in model pairs, and iii) the primary purpose of employing diverse model pairs across tasks was to test the robustness of our findings.

We already provide some evidence on the potential importance of the model pair's characteristics in Section 4.2.<sup>30</sup> Table 3 investigates whether participants select models based on characteristics of either models or model pairs that are independent of the signal. Looking at columns (4) and (5), we concluded that such signal-independent characteristics do not play a major role in our setting. Hence, model pair's characteristics that are independent of the signal cannot explain well which model participants select, conditional on model selection. However, they may still influence i) which updating rule participants use more generally, and ii) the likelihood of making mistakes in applying a model selection criterion.

To examine these questions, we replicate the classification of reported guesses presented in the main text for the five different model pairs. Table B1 and Table B2 give the classification for each model pair. Table B1 requires an exact match between the predictions of the updating rules and the guesses, while Table B2 allows for a distance between guesses and predictions of at most 2 percentage points. Additionally, Figure B2 presents the estimated model weights for the different model pairs, and Figures B3 and B4 present the distribution of the weights conditional on the pair and the observed signal.

When considering our main four point predictions, we find the classification of the reported guesses and estimated model weights are strikingly similar across model pairs and consistent with the pooled data presented in the main text. For example, in Table B2, we observe that the share of observations corresponding to one-stage updating and model selection ranges from 15% to 21% and 44% to 47%, respectively. Bayesian updating accounts for approximately 5% of all observations, except for Model Pair 5, where we observe a substantially higher proportion of Bayesian updating; however, this outlier can be explained by the overlap between the predictions of the Bayesian and one-stage rules for this model pair. These findings suggest that model pair characteristics do not play a major role in determining which updating rules individuals use. This aligns with the observation that individuals typically apply updating rules consistently across tasks (see Section 4.4) and that they rarely employ multiple different rules (see Section D.4).

<sup>&</sup>lt;sup>30</sup>The analysis reported in the main text focuses primarily on the characteristics of model pairs defined conditional on the observed signal, such as fit and informativeness. Our findings indicate that the comparison between the fit levels, rather than informativeness, plays a major role in model selection.

When focusing on participants who engage in model selection, we find that the bestfitting model is selected about 75% of the time. However, Model Pair 4 shows some deviations from this pattern. In this model pair, participants choose the worst-fitting model more frequently than the best-fitting one: only 39% of guesses correspond to the best-fitting model.<sup>31</sup> Model Pair 4 differs substantially from other model pairs as it features a fully uninformative model (equal number of colored balls in each bag) and a fully informative model (each bag contains only balls of a specific color).<sup>32</sup> However, we do not find evidence that participants are systematically drawn to the fully informative nor the fully uninformative model, as shown in Table 3. Therefore, the frequent selection of the worst-fitting model cannot be attributed to the informativeness of the models or any characteristics independent of the signal. While we cannot conclusively explain the pattern in Model Pair 4 and acknowledge that further investigation is needed, the most plausible explanation, also considering our other findings, appears to be based on mistakes in evaluating the fit of models.

Finally, we will explore whether participants are more likely to select the worst-fitting model when the models are close in fit, making it potentially more challenging to identify the best-fitting model. There are some differences in fit levels across model pairs that can be utilized for this purpose, although these differences are relatively small: Model Pair 2 has the smallest fit difference (0.167), followed by Model Pair 5 (0.25), and then Model Pair 1, Model Pair 3, and Model Pair 4 (0.33). However, as Table B2 shows, these differences do not align with the observed variation in the share of selecting the worst-fitting model across tasks.<sup>33</sup>

 $<sup>^{31}</sup>$ The share of guesses consistent with the best-fitting model does not vary significantly across signals (conditional on model selection: 40% given the orange signal and 38% given the purple signal, p-value = 0.8419; not conditional on model selection: 16% given the orange signal and 20% given the purple signal, p-value = 0.4775).

<sup>&</sup>lt;sup>32</sup>Moreover, the models are also not conflicting, as defined in Section 2, because the fully uninformative model always implies a posterior equal to the prior. However, the posterior vector resulting from maximum likelihood selection would be Bayes-inconsistent.

<sup>&</sup>lt;sup>33</sup>When we regress a dummy for selecting the best-fitting model on the difference in fit levels between proposed models, excluding observations that do not correspond to model selection (within 2 p.p.), we find a statistically significant relationship (N=956, coefficient = -0.675, t = -3.62, p < 0.001). However, Model Pair 4 stands out with a disproportionately large share of potential mistakes. When we exclude observations from Model Pair 4, the relationship is no longer statistically significant (N=815, coefficient = -0.043, t = -0.23, p = 0.818). This suggests that other characteristics of Model Pair 4 might explain the frequent potential mistakes. Ultimately, we cannot identify the features that lead to differential updating for this model pair, but we acknowledge that proposing a pair of such distinct models—one fully informative and the other uninformative—appears to be more complex for participants.





Notes. In all figures, the black circle corresponds to the prior over the state, the blue diamonds to the two model predictions, the purple square to the Bayesian updating for  $Pr(m_1) = 50\%$ , the red square to the one-stage updating, the blue square to the prediction by selecting the best-fitting models, and the orange square to the prediction by selecting the worst-fitting models; the gray areas represent the Bayes-consistent vector of posteriors. We also indicate the prediction by selecting the most informative model given each signal.



Figure B2: Estimated Model Weight

Notes. The figure plots the distribution of weights on the best-fitting model,  $\rho$ , for pooled data and for each model pair. We report the implied weights for the reported guesses within the model predictions in blue. The red distribution serves as a benchmark, illustrating how the weights should be distributed if participants follow Bayesian updating in all tasks. The last bar on the right corresponds to the share of reported posteriors outside model predictions for which we cannot recover model weights.



Figure B3: Estimated Model Weight (Orange Signal)

Notes. The figure plots the distribution of weights on the best-fitting model,  $\rho$ , for pooled data and for each model pair, conditional on an orange signal. We report the implied weights for the reported guesses within the model predictions in blue. The red distribution serves as a benchmark, illustrating how the weights should be distributed if participants follow Bayesian updating in all tasks. The last bar on the right corresponds to the share of reported posteriors outside model predictions for which we cannot recover model weights.



Figure B4: Estimated Model Weight (Purple Signal)

Notes. The figure plots the distribution of weights on the best-fitting model,  $\rho$ , for pooled data and for each model pair, conditional on a purple signal. We report the implied weights for the reported guesses within the model predictions in blue. The red distribution serves as a benchmark, illustrating how the weights should be distributed if participants follow Bayesian updating in all tasks. The last bar on the right corresponds to the share of reported posteriors outside model predictions for which we cannot recover model weights.

Type of Guess	Model Pair 1	Model Pair 2	Model Pair 3	Model Pair 4	Model Pair 5
Bayesian	2.67	4.00	2.83	2.33	2.67
One-stage	11.00	10.17	9.50	14.33	12.00
Best-fitting	32.00	30.83	30.83	17.33	29.33
Worst-fitting	8.33	7.33	8.00	27.00	10.67
Within	31.33	36.17	34.50	33.33	32.33
Outside	14.67	11.50	14.33	5.67	13.00
Total	100.00	100.00	100.00	100.00	100.00

Table B1: Classification of Guesses by Model Pair: Exact %

*Notes.* Each column reports the shares of guesses that can be exactly classified as one of the point predictions (Bayesian, one-stage, best-fitting, and worst-fitting) or in one of the two residual categories ("Within" if the guess is within the two model predictions or "Outside" otherwise) for each model pair.

Type of Guess	Model Pair 1	Model Pair 2	Model Pair 3	Model Pair 4	Model Pair 5
Bayesian	6.00	8.83	4.67	5.33	18.67
One-stage	20.33	16.50	15.50	18.33	23.33
Best-fitting	35.67	36.50	36.33	18.33	32.00
Worst-fitting	10.00	9.00	9.33	28.67	11.67
Total	72.00	68.83	65.83	70.66	73.34

Table B2: Classification of Guesses by Model Pair: Within 2 p.p.

*Notes* Each column reports the shares of guesses that can be classified as one of the point predictions (Bayesian, one-stage, best-fitting, and worst-fitting) or in one of the two residual categories ("Within" if the guess is within the two model predictions or "Outside" otherwise) for each model pair, allowing a distance of at most 2 p.p. between the prediction and the reported guess. Note that the total does not double-count the overlap of guesses classified both as Bayesian and One-stage: 0% for Model Pair 1, 3, and 4; 2% for Model Pair 2; and 10% for Model Pair 5.

## C Appendix: Calibrating a Stochastic Model of Model Selection

This section explores how well our data aligns with a stochastic model of model selection via maximum likelihood. To do so, we estimate the parameters of a simple stochastic model of model selection that includes a heterogeneous population applying different model selection criteria.

Suppose a DM uses one of the criteria for model selection discussed in Section 2: maximum likelihood, informativeness, or dogmatic model selection. The shares of DMs using each criterion are represented by  $s_{best}$ ,  $s_{info}$ , and  $s_{dogm}$ , respectively. Also, some DMs could consistently choose the worst-fitting model, with the share  $s_{worst}$ . We allow DMs to make errors when applying these criteria, e.g., when comparing the model fit levels. With probability  $1 - \varepsilon$ , the DM selects the model according to their criterion, but with probability  $\varepsilon$ , they make a mistake and select the other model. We assume  $\varepsilon$  is constant across participants and updating tasks. This simplifying assumption enables us to perform this exercise and we do not find compelling reasons to adopt a different assumption.

Note that distinguishing between the rate of mistakes  $\varepsilon$  and the model selection rules is challenging. For example, any posterior belief that corresponds to the prediction of one of the two models can be explained by stochastic versions of model selection based on maximum likelihood or model selection based on informativeness. Furthermore, dogmatic selection imposes minimal structure on individual guesses, allowing it to explain any such posterior without assuming randomness. However, by exploiting the repeated updating tasks, we can identify both s<sub>best</sub> and  $\varepsilon$ .

First, we can identify  $\varepsilon$  from repeated updating tasks when participants observe the same signal twice (see Section 4.1). Participants who choose different models in these two tasks have made one mistake. In our simple model, the proportion of such cases, s<sub>diff</sub>, corresponds to:

$$s_{\text{diff}} = 2 \cdot \varepsilon \cdot (1 - \varepsilon).$$

We estimate  $s_{diff}$  from our data, focusing on the 118 observations where a model selection rule was used in both tasks. By using the proportion of participants who select different models in the two updating tasks in the Table A5, we calculate that  $s_{diff} = \frac{2.67+3.67}{28.67+2.67+3.67+4.33} = 0.161$ . Hence, we estimate  $\varepsilon$  to be 0.0883.

Next, using this estimated  $\varepsilon$  value, we can proceed to estimate the shares of DMs following different model selection criteria by using the repeated model pairs when participants observe different signal realizations. Recall from Section 2 that the maximum likelihood criterion requires the selection of the best-fitting model for both signal realizations, which is a different model for the two signals; the same applies to selecting the worst-fitting model. Instead, according to the dogmatic criterion, the participant should select the same model for both signals. Finally, according to the informativeness criterion, participants should select Model 2 for both signals in Model Pair 2 and Model 1 for both signals in Model Pair 3. Note that dogmatic updating imposes minimal structure on model selection, and for the repeated model pairs we used in our experiment, the dogmatic and informativeness criteria overlap, and thus, we cannot distinguish between them without strong assumptions. Since our main interest is in estimating  $s_{best}$ , we combine these criteria under  $s_{same} = s_{info} + s_{dogm}$ .

We aim to identify the proportion of participants who, in principle, follow a specific criterion but make mistakes, based on the share of participants who report certain vectors of posterior beliefs. We denote the proportions of participants who chose the best-fitting model, the worst-fitting model, and the same model for both signals as  $\hat{s}_{\text{best}}, \hat{s}_{\text{worst}}$ , and  $\hat{s}_{\text{same}}$ , respectively. Our simple model posits that these observable shares are as follows:

$$\hat{s}_{\text{best}} = (1 - \varepsilon)^2 \cdot s_{\text{best}} + \varepsilon^2 \cdot s_{\text{worst}} + (1 - \varepsilon) \cdot \varepsilon \cdot s_{\text{same}},$$
$$\hat{s}_{\text{worst}} = \varepsilon^2 \cdot s_{\text{best}} + (1 - \varepsilon)^2 \cdot s_{\text{worst}} + (1 - \varepsilon) \cdot \varepsilon \cdot s_{\text{same}},$$
$$\hat{s}_{\text{same}} = 2 \cdot (1 - \varepsilon) \cdot \varepsilon \cdot (1 - s_{\text{same}}) + (\varepsilon^2 + (1 - \varepsilon)^2) \cdot s_{\text{same}}.$$

Because  $\hat{s}_{\text{best}} + \hat{s}_{\text{worst}} + \hat{s}_{\text{same}} = 1$  and  $s_{\text{best}} + s_{\text{worst}} + s_{\text{same}} = 1$ , we have a system of two equations in two unknowns. We calculate the different shares from our data by focusing on participants who select a model in both updating tasks (see Table 4, Column "Within 2 p.p."). We find that  $\hat{s}_{\text{best}} = \frac{23.33}{5.33+6+23.33+2.67} = 0.625$  and  $\hat{s}_{\text{same}} = \frac{5.33+6}{5.33+6+23.33+2.67} = 0.3035$ . Based on these estimates, we calculate the following shares of participants following the different model selection criteria:  $s_{\text{best}} = 0.7310$ ,  $s_{\text{worst}} = 0.0588$ , and  $s_{\text{same}} = 0.2102$ .

Therefore, using this simple calibration exercise, we conclude that model selection can be well described by a stochastic model of model selection via maximum likelihood, with a small minority of participants using other selection rules. While this calibration exercise is based on a very simple model, we believe that it provides a reliable rough estimate of the share of participants who engage in model selection via maximum likelihood, when accounting for stochasticity.

## D Appendix: Other Updating Rules

In the main test, we focus on four updating rules: Bayesian updating, one-stage updating, model selection, and reporting the prior over the state. We find that these updating rules can capture most of our data. For individual guesses, Table 2 shows that 53.81% of all reported guesses exactly match these predictions, and this share increases to 71.67% when allowing a distance of at most 2 p.p between the prediction and the reported guess. Moreover, Table 4 illustrates that between 41.67% and 59.01% of the reported vectors of posteriors are consistent with these updating rules. Finally, we also find that participants consistently use these rules: 40.34% of participants use the same updating rule for all 7 guesses, and another 13.66% use the same rule for 5 or 6 out of the 7 tasks (Table 5).

In the following, first, we study whether participants systematically use other updating rules, despite the little margin left by the fact that the rules mentioned above capture most guesses. Then, we also examine whether some participants use multiple updating rules across tasks.

#### D.1 Over- and Underinference about the Models

In this section, we explore the possibility that some participants report beliefs that correspond to over- and underinference about the models. We discuss in Section 2, how model selection via maximum likelihood and one-stage updating can be interpreted as extreme forms of such biases in inference about the models. Here, we investigate less extreme forms of over- or underinference about the models, both theoretically and empirically.

#### D.1.1 Theoretical Framework

To more broadly capture over- and underinference about the models, we express the weight placed on the model m as

$$\rho_s^m = \frac{1}{1 + \left(\frac{\Pr(s|m')}{\Pr(s|m)}\right)^{\alpha}},\tag{D2}$$

where the parameter  $\alpha$  captures different type of biased inference:  $\alpha = 1$  corresponds to Bayesian updating;  $\alpha > 1$  corresponds to overinference with special case  $\alpha \to \infty$  as model selection via maximum likelihood;  $\alpha \in [0,1)$  corresponds to underinference with special case  $\alpha = 0$  as one-stage updating; and  $\alpha < 0$  implies updating in the wrong direction. Finally, selecting the worst-fitting model corresponds to  $\alpha \to -\infty$ . Note that this general approach to the introduction of biases is related to the method proposed by Grether (1980) to capture biases in updating beliefs about state.

Figure D1 illustrates how vectors of posteriors can be classified as reflecting over- or underinference about the models. A DM exhibiting overinference (underinference) about the models for both signals would report a vector of posteriors in the blue (red) area. The colored lines correspond to the predicted beliefs for different levels of  $\alpha$  in Equation D2, assuming a constant  $\alpha$  across signals. The main updating rules that we consider are special cases of different biases in inference about the models: best-fitting for overinference, one-stage for underinference, and worst-fitting for inference in the wrong direction.<sup>34</sup> Note that over- and underinference about the models imply that the reported posteriors always fall within the range of the model predictions.



Figure D1: Predictions for Vectors of Posteriors: Biases in Model Weighting

Notes. The figures illustrate two models with the following parameters:  $\Pr^{m_1}(p|A) = 1/6$ ,  $\Pr^{m_1}(p|B) = 3/6$ ,  $\Pr^{m_2}(p|A) = 5/6$ , and  $\Pr^{m_2}(p|B) = 1/6$ . In both figures, the black circle corresponds to the prior over the state, the blue diamonds to the two model predictions, the purple square to Bayesian updating with  $\Pr(m_1) = 50\%$ , the red square to the one-stage updating, the blue square to selecting the best-fitting model, and the orange square to selecting the worst-fitting model; the gray areas represent the Bayes-consistent vectors of posteriors. The colored areas represent the classification for over- and underinference for both signals (red), overinference for a signal and underinference for the other signal (purple), wrong direction for both signals (yellow), overinference for a signal and wrong direction for the other signal (orange). The colored lines capture overinference (blue), underinference (red), and wrong direction (yellow) for a constant degree of  $\alpha$  across signals as in Equation D2.

We can generalize this framework to allow for mistakes in evaluating the fit levels of the competing models. If the DM makes a mistake of this type, they sometimes identify the wrong model as the best-fitting one. Therefore, the weight placed on the model m can be expressed more generally as:

$$\rho_s^m = \frac{1}{1 + \left(\frac{\Pr(s|m')}{\Pr(s|m)}\right)^{\lambda\alpha}},\tag{D3}$$

where the parameter  $\lambda \in \{1, -1\}$  determines whether the DM is subject to these mistakes. When  $\lambda = 1$ , D3 corresponds to deterministic model without mistakes. However, in the stochastic model, with a certain probability, the DM makes an error, resulting in  $\lambda = -1$ .

 $<sup>^{34}</sup>$ Note that model selection based on informativeness and dogmatic model selection are not part of this class of updating rules.

When  $\alpha = \infty$ , D3 represents the stochastic version of model selection via maximum likelihood, as discussed in Section C.

#### D.1.2 Results

We begin our analysis by focusing on the deterministic case, where there are no mistakes in identifying the best-fitting model ( $\lambda = 1$ ). Our experimental design allows us to identify  $\alpha$  from the individual guesses. When pooling all individual guesses, we find that the most frequent bias is overinference about models, with 43% of guesses falling into this category. Furthermore, we find that 17% of guesses are classified as underinference, while 24% represent inference in the wrong direction. The remaining observations either correspond to Bayesian updating or fall outside the range of the model predictions.

In the main text, we primarily focuse on parameters  $\alpha = 1$  (Bayesian updating),  $\alpha = 0$  (one-stage updating), and  $\alpha \to \infty$  (selecting the best-fitting model) or  $\alpha \to -\infty$  (selecting the worst-fitting model). While we have shown that these specific values account for much of the data, here we explore the possibility that some participants consistently report beliefs consistent with other values of  $\alpha$ . Figure D2 and Figure D3 present the distribution of  $\alpha$  estimated from individual guesses across all tasks and the ones estimated from the vectors of posteriors for the repeated model pairs. These figures do not reveal any frequently used  $\alpha$  values other than the ones associated with the main updating rules:  $\alpha = 0$  (one-stage updating),  $\alpha = 1$  (Bayesian updating),  $\alpha = \infty$  (selecting the best-fitting model), and  $\alpha = -\infty$  (selecting the worst-fitting model).

Next, we study whether the reported beliefs of some participants can be consistently described by a certain value of  $\alpha$ . Therefore, we focus on individuals who do not systematically use any of the main updating rules discussed so far (Bayesian or one-stage updating, model selection, or reporting the prior). Specifically, we consider the 138 participants who used each of these rules in at most 4 tasks, as shown in Table 5. To account for some instability of  $\alpha$ , we consider three broad classifications: overinference about the models captured by  $\alpha \in (1, \infty)$ , underinference about the models captured by  $\alpha \in (0, 1)$ , and inference about the models in the wrong direction captured by  $\alpha \in (-\infty, 0)$ . Table D1 presents the distribution of the frequency with which participants use these different updating rules in the seven updating tasks. It indicates that none of these three broad categories is used consistently.

However, the analysis so far assumed that individuals always correctly identify the bestfitting model. Next, we focus on the stochastic case ( $\lambda \in \{1, -1\}$ ). We again consider the 138 participants who use Bayesian updating, one-stage updating, model selection or report the prior in at most 4 tasks. To account for mistakes in identifying the bestfitting model, as well as some instability of  $\alpha$ , we consider two broad classifications: overinference about the models captured by  $\lambda \alpha \in (-\infty, -1) \cup (1, \infty)$  and underinference about the models captured by  $\lambda \alpha \in (-1, 1)$ . Table D2 presents the distributions of participants consistent with these categories. When requiring participants to remain within the same category across all seven guesses, we classify 4.35% of the remaining sample as overinferring about models and 0.72% as underinferring about models. By allowing participants to deviate from the category in one or two guesses, we classify 15.93% (22 participants) and 34.05% (47 participants) of the remaining participants, respectively. This corresponds to 7.33% and 15.66% of the full sample of 300 participants. These findings suggest that some participants consistently use less extreme forms of over-or underinference about models but occasionally make errors in identifying the best-fitting model.





Notes. Cumulative distribution of  $\alpha$  across tasks. We exclude observations for which reported guesses fall outside the range of the two model predictions. Note that almost all observations with  $\alpha < -10$  and  $\alpha^m > 10$  correspond to selecting the worst- or best-fitting model, respectively ( $\alpha = -\infty$  or  $\alpha = \infty$ ).



Figure D3:  $\alpha$  in the vectors of Posteriors

Notes. The size of the circles is relative to the number of observations. Data are in steps of 1/3. For this figure, we code selecting the best-fitting or worst-fitting model, corresponding to  $\alpha = \infty$  and  $\alpha = -\infty$ , as 11 and -11.

Nr. Consistent	$c \in (0, 1)$	$\alpha \in (1, \infty)$	$\alpha \in (-\infty, 0)$
Observations	$\alpha \in (0, 1)$	$\alpha \in (1,\infty)$	$\alpha \in (-\infty, 0)$
0	49.28	15.22	21.74
1	35.51	30.43	28.99
2	12.32	24.64	27.54
3	2.17	15.94	11.59
4	0.00	6.52	7.97
5	0.00	6.52	2.17
6	0.72	0.72	0.00
7	0.00	0.00	0.00
Total	100.00	100.00	100.00

Table D1: Consistency of Updating Rules: Less Extreme Forms of Over- and Underinference about the Models, Deterministic Case

Notes. The table reports the frequencies of different individuals using specific updating rules. Out of the 7 guesses reported by each participant, we report how many times each rule is used (Column "Nr. Consistent Observations"). The columns display the distribution of frequencies for different levels of  $\alpha$ . For example, the column " $\alpha \in (0, 1)$ " shows the share of participants who report guesses that correspond to any  $\alpha \in (0, 1)$  in 0, 1, 2, 3, 4, 5, 6, and 7 tasks. The sample consists of the 138 participants who use one of the updating rule (Bayesian, one-stage, model selection, prior) in at most 4 out of 7 tasks, allowing for a distance of at most 2 p.p. between the prediction and the reported guess (see Table 5).

Nr. Consistent	$\sum_{i=1}^{n} c_i (1, 2\alpha) = 1 + (1, 2\alpha)$	$\sum c (11)$
Observations	$\lambda a \in (-\infty, -1) \cup (1, \infty)$	$\lambda \alpha \in (-1, 1)$
0	1.45	18.12
1	8.70	34.78
2	14.49	16.67
3	26.09	16.67
4	22.46	6.52
5	12.32	5.80
6	10.14	0.72
7	4.35	0.72
Total	100.00	100.00

Table D2: Consistency of Updating Rules: Less Extreme Forms of Over- and Underinference about the Models, Stochastic Case

Notes. The table reports the frequencies of different individuals using specific updating rules. Out of the 7 guesses reported by each participant, we report how many times each rule is used (Column "Nr. Consistent Observations"). The columns display the distribution of frequencies for different levels of  $\lambda \alpha$ . For example, the column " $\lambda \alpha \in (-\infty, -1) \cup (1, \infty)$ " shows the share of participants who report guesses that correspond to any  $\lambda \alpha \in (-\infty, -1) \cup (1, \infty)$  in 0, 1, 2, 3, 4, 5, 6, and 7 tasks. The sample consists of the 138 participants who use one of the updating rule (Bayesian, one-stage, model selection, prior) in at most 4 out of 7 tasks, allowing for a distance of at most 2 p.p. between the prediction and the reported guess (see Table 5).

#### D.2 Signal-independent Model Weights

This section explores the possibility that some participants use model weights that are independent of signals. Examples of such updating rules are dogmatic model selection and one-stage updating. We expand our empirical investigation to consider a broader class of updating rules with this feature, guided by a theoretical framework to capture such updating.

#### D.2.1 Theoretical Framework

Our goal is to capture the broad class of updating rules in which the model weights are independent of the signal. Such a DM places weight  $\rho_s^m = \rho_{s'}^m = c \in [0, 1]$  on one model, m, and weight  $\rho_s^{m'} = \rho_{s'}^{m'} = 1 - c$  on the other model, m'. This class of rules includes dogmatic model selection with  $c \in \{0, 1\}$  and one-stage updating with c = 0.5.

Building on the graphical representation introduced in Section 2, the green line in Figure D4 illustrates the vectors of posteriors for different values of c. Other updating rules, such as Bayesian updating, predict that  $\rho_s^m$  is not signal-independent but depends on the fit levels of the two models conditional on the observed signal s. This also applies to any other updating rules that reveal under- or overinference about the models, as in Section D.1, with the exception of one-stage updating. One-stage updating can be described in terms of both underinference and of signal-independence ( $\alpha = 0$  and c = 0.5, respectively).

Note that our main analysis is described in terms of the weight placed on the best-fitting model,  $\rho$ . In this case, we should expect its value to be in  $\{c, 1-c\}$ .<sup>35</sup>

#### D.2.2 Results

Our experimental design allows us to identify the model weights for the reported posteriors that fall within the model predictions. As in the main text, we denote with  $\rho$  the weight a participant assigns to the best-fitting model. In the following, we explore the possibility that some participants consistently weight models with  $\rho \in \{c, 1 - c\}$ .

Figure 3 presents the overall distribution of  $\rho$  estimated from the individual guesses across all tasks. This figure does not reveal any frequently used  $\rho$  values other than the ones associated with the main updating rules:  $\rho = 0$  (selecting the worst-fitting model),  $\rho = 0.5$  (one-stage updating), and  $\rho = 1$  (selecting the best-fitting model).

<sup>&</sup>lt;sup>35</sup>More precisely, signal-independence implies that  $\rho = c$  for one signal and  $\rho = 1 - c$  for the other signal, as the best-fitting model always varies across signals. The condition  $\rho \in \{c, 1 - c\}$  also allows for  $\rho = c$  across both signals, which would not reflect signal-independence, but capture over- and underinference about the models as examined in Section D.1, though with a slightly different structural form. This approach would also capture any posteriors lying on the straight line connecting the "best-fitting" and "worst-fitting" points in Figure D4. It is possible to empirically distinguish between these two classes of updating rules only with data on the reported vectors of posteriors. In our empirical approach, we do not differentiate between these two classes of rules, instead considering a broader class of updating rules described by  $\rho \in \{c, 1 - c\}$ . We then present evidence against the relevance of this broader class, providing strong evidence against signal-independent model weighting.



Figure D4: Predictions for Vectors of Posteriors

Notes. The figures illustrate two models with the following parameters:  $\Pr^{m_1}(p|A) = 1/6$ ,  $\Pr^{m_1}(p|B) = 3/6$ ,  $\Pr^{m_2}(p|A) = 5/6$ , and  $\Pr^{m_1}(p|B) = 1/6$ . The black circle corresponds to the prior over state, the blue diamonds to the two model predictions, the purple square to Bayesian updating for  $\Pr(m_1) = 50\%$ , the red square to the one-stage updating, the blue square to selecting the best-fitting model, and the orange square to selecting the worst-fitting models; the gray areas represent the Bayes-consistent vector of posteriors. The green lines capture vectors of posterior resulting from updating rules where model weights are independent of the signals, for all values of c.

Note, however, that signal-independence does not require  $\rho$  to be constant; rather, it requires  $\rho \in \{c, 1 - c\}$ . To examine whether there are any frequently used c values, we can analyze the distribution of  $|\rho - 0.5|$ . A DM who weights models according to  $\rho \in \{c, 1 - c\}$  would exhibit a constant  $|\rho - 0.5|$  across updating tasks. Figure D5 shows the distribution of  $|\rho - 0.5|$ , revealing that the only frequent values are 0 (one-stage updating) and 0.5 (selection of the best- or worst-fitting model). Thus, we conclude that there are no other frequent c values consistently used by participants.

Finally, we study whether the reported beliefs of some participants can be consistently described by a value of c such that  $\rho \in \{c, 1-c\}$ . Therefore, we focus on individuals who do not systematically use any of the main updating rules discussed so far (Bayesian or one-stage updating, model selection, prior). Specifically, we consider the 138 participants who used each of these updating rules in at most 4 tasks, as shown in Table 5.

To account for  $\rho \in \{c, 1-c\}$  and allow for some instability of  $\rho$ , we consider six broad categories:  $\rho \in [0, 0.05] \cup (0.95, 1], \rho \in (0.05, 0.15] \cup (0.85, 0.95], \rho \in (0.15, 0.25] \cup (0.75, 0.85], \rho \in (0.25, 0.35] \cup (0.65, 0.75], \rho \in (0.35, 0.45] \cup (0.55, 0.65], \text{ and } \rho \in (0.45, 0.55].$  Table D3 presents these results, indicating that none of these six broad categories is used consistently. This finding suggests that participants do not consistently use a constant  $\rho$  or  $\rho \in \{c, 1-c\}$  other than the ones discussed in the main text.



Figure D5: Distribution of  $|\rho-0.5|$  Notes. The figure plots the distribution of  $|\rho-0.5|$ 

Nr. Consistent Observations	$\rho \in [0, 0.05] \cup (0.95, 1]$	$\rho \in (0.05, 0.15] \cup (0.85, 0.95]$	$\rho \in (0.15, 0.25] \cup (0.75, 0.85]$	$\rho \in (0.25, 0.35] \cup (0.65, 0.75]$	$\rho \in (0.35, 0.45] \cup (0.55, 0.65]$	$\rho \in (0.45, 0.55]$
0	53.62	51.45	47.10	38.41	34.78	46.38
1	18.12	29.71	25.36	32.61	36.96	26.81
2	15.94	14.49	19.57	19.57	15.94	17.39
3	7.97	2.90	6.52	8.70	6.52	6.52
4	4.35	0.72	1.45	0.72	3.62	2.90
5	0.00	0.72	0.00	0.00	2.17	0.00
6	0.00	0.00	0.00	0.00	0.00	0.00
7	0.00	0.00	0.00	0.00	0.00	0.00
Total	100.00	100.00	100.00	100.00	100.00	100.00

Table D3: Consistency of Updating Rules: Signal-independent Model Weights

Notes The table reports how often different participants use specific updating rules. Since participants complete 7 updating tasks, they can use each rule between 0 and 7 times (Column "Nr. Consistent Observations"). The columns then display the distribution of frequencies for different levels of  $\rho$ . For example, the column " $\rho \in (0.05, 0.15] \cup (0.85, 0.95]$ " shows the share of participants who report guesses that correspond to any  $\rho \in (0.05, 0.15]$  or  $\in (0.85, 0.95]$  in 0, 1, 2, 3, 4, 5, 6, and 7 tasks.

#### D.3 Biases based on the Compound Model

We study belief updating in the presence of competing models by focusing on how individuals weight models to combine their predictions over state. Section 2 presents this approach and the main biases in model weighting that we consider in this project. However, instead of aggregating the predictions of the different models, a DM could first aggregate the competing models into a single model and then use this model to update their beliefs given the observed signal. For a Bayesian DM, these two approaches are equivalent if this compound model is derived by combining the models using the prior over models, Pr(m). However, as we discuss in this section, biases in updating given such a compound model lead to fundamentally different deviations from the Bayesian benchmark than biases in model weighting as discussed in Section 2. Most importantly, biases based on the compound model reflect the updating biases studied in earlier work (e.g., Benjamin, 2019) and cannot explain the systematic emergence of Bayes-consistencies. For simplicity, we refer to such biases as "biases in updating about the state," distinguishing them from "biases in updating about the models," which are the main focus of our paper.

In this section, we first present a theoretical framework examining biases in updating given the compound model and then study whether this alternative approach can explain the patterns observed in our data. The main goal of this section is to test the descriptive validity of such an approach and to demonstrate that biases in model weighting are empirically distinct from those resulting from updating based on a single model.

#### D.3.1 Theoretical Framework

In the presence of competing models, a DM responding to a signal could either combine the predictions of the different models, as discussed in Section 2, or aggregate the models into a single model and then use this to update their beliefs. This section focuses on the latter option.

We begin by illustrating that these two approaches are equivalent for a Bayesian DM. In Section 2, we describe the posterior beliefs of a Bayesian DM can be described as:

$$\Pr(A|s) = \rho_s^{m_1} \Pr^{m_1}(A|s) + \rho_s^{m_2} \Pr^{m_2}(A|s),$$
(D4)

where the model weight,  $\rho_s^m$ , corresponds to  $\Pr(m)\Pr(s|m)/\Pr(s)$ , starting from prior over the models  $\Pr(m)$ . Equivalently, the same posterior beliefs could be calculated as:

$$\Pr(A|s) = \frac{\Pr(A) \,\Pr^{\bar{m}}(s|A)}{\Pr^{\bar{m}}(s)} \tag{D5}$$

where  $\Pr^{\bar{m}}(s|\omega) = \Pr(m_1) \Pr^{m_1}(s|\omega) + \Pr(m_2) \Pr^{m_2}(s|\omega)$ , for each  $\omega$  and  $\Pr^{\bar{m}}(s) = \Pr(A)\Pr^{\bar{m}}(s|A) + \Pr(B)\Pr^{\bar{m}}(s|B)$ . The latter equation allows for an alternative interpretation: the Bayesian DM revises their beliefs via Bayes' rule by using a *compound model*,  $\bar{m}$ , derived by combining the two models using the prior over the models.

However, a DM may be biased when using the compound model to incorporate the signal into their beliefs. Using Equation D5, it is straightforward to introduce the classical biases in updating when encountering one model. Using the classical approach of Grether (1980), we can capture such biases as:

$$\hat{\Pr}(A|s) = \frac{1}{1 + \left(\frac{\Pr^{\bar{m}}(s|B)}{\Pr^{\bar{m}}(s|A)}\right)^{\alpha_s}},\tag{D6}$$

where  $\alpha_s = 1$  corresponds to Bayesian updating,  $\alpha_s \in [0, 1)$  to underinference about the states,  $\alpha_s > 1$  to overinference about the states, and  $\alpha_s < 0$  to inference in the wrong direction. This classification describes mistakes associated with the misperceived diagnosticity of the observed signal according to the reduced model. For example, underinference represents the case in which the reported posterior is closer to the prior compared to the Bayesian benchmark and thus is "as if" the DM responds to a less diagnostic signal than the observed one. Figure D6 illustrates how these biases could classify different vectors of posterior beliefs, either by allowing  $\alpha_s$  values to differ for various signals (colored areas) or by maintaining a constant level of  $\alpha_s$  across signals (colored lines).

The comparison between Figure 1b and Figure D6 highlights important differences between biases in model weighting and biases in updating about the state given the compound model. First, even incorporating biases in model weighting, the biased posteriors always fall within the range of the two model predictions. In contrast, biases in updating about the state can result in posteriors that lie outside this range. Second, the predictions of our main updating rules—selecting the best-fitting or worst-fitting model and one-stage updating—cannot be captured by this framework, with the except of Bayesian updating. The predictions from these updating rules not only do not correspond to any specific values of  $\alpha_s$ , but they also correspond to inconsistent biases across signal realizations. For example, selecting the best-fitting model would require overinference about the state conditional on one signal and inference in the wrong direction conditional on the other signal. Third, and most importantly, the figure illustrates that biases in updating about the state result in Bayes-consistent vectors of posteriors. This prediction does not depend on individuals using a stable parameter of  $\alpha_s$  across signals but rather any combination of  $\alpha_s \geq 0$  across signals, that is, when they update in the right direction even if in a biased way. In particular, Bayes-inconsistencies are only predicted when a DM updates in the wrong direction ( $\alpha_s < 0$ ) for only one of the two signals. Such updating does not reflect a consistent updating rule, it should only happen infrequently. As a benchmark, note that a recent study where participants encounter only one model (Aina et al., 2024) finds that the share of Bayes-inconsistent posteriors is quite low (6%). This prediction of biases in updating about the state stands in stark contrast to biases in model weighting, which predict frequent and systematic Bayes-inconsistencies for updating rules such as model selection via maximum likelihood.

Note that the DM may also aggregate the competing models into a single model in a



Figure D6: Predictions for Vectors of Posteriors

Notes. The figures illustrate two models with the following parameters:  $\Pr^{m_1}(p|A) = 1/6$ ,  $\Pr^{m_1}(p|B) = 3/6$ ,  $\Pr^{m_2}(p|A) = 5/6$ , and  $\Pr^{m_1}(p|B) = 1/6$ . The black circle corresponds to the prior over the state, the blue diamonds to the model predictions, the purple square to Bayesian updating with  $\Pr(m_1) = 50\%$ , the red square to one-stage updating, the blue square to selecting the best-fitting model, and the orange square to selecting the worst-fitting model; the gray areas represent the Bayes-consistent vector of posteriors. The colored areas represent the classification for over- and underinference, respectively: overinference for both signals (blue), underinference for both signals (red), overinference for a signal and underinference for the other signal (purple), wrong direction for both signals (yellow), overinference for a signal and wrong direction for the other signal (green), and underinference (blue), underinference (red), and wrong direction (yellow) for a constant degree of  $\alpha_s$  across signals as Equation D6.

biased manner, which could introduce an additional layer of bias that we do not consider in the analysis above. However, crucially, in this case, the DM still only entertains one model and all insights discussed in the previous section still apply. Most importantly, this approach cannot capture Bayesian inconsistencies or updating rules that involve strong over- or underinference about models.

#### D.3.2 Results

In the main text, we document that a rather small share of participants update their beliefs according to Bayes' rule, which aligns with both biases in model weighting and biases in updating based on a compound model. However, three findings from Section 4 provide evidence in contrast with the idea that our data can be well explained by participants first combine the models into one and then using the latter to update beliefs. Note that these findings not only challenge the framework introduced in Section D.3.1 but also the broader use of a single model to update beliefs. First, we observe that most guesses fall within the range of the two model predictions, which is not predicted by biases in updating about the state. Only 8% of all reported posteriors fall outside the model predictions by more than 2 percentage points. Second, biases in updating about the state can hardly explain why the posteriors of most participants correspond to either model selection or one-stage updating; there are no reasons to expect such belief patterns to emerge. For example, in the context of the framework of Section D.3.1, such updating would correspond to unstable values of  $\alpha_s$ . The frequency of model selection via maximum likelihood is particularly inconsistent with updating based on a single model, as it would require participants to frequently update in the wrong direction, as discussed in Section D.3.1. Third, we find that half of the reported vectors of posteriors are Bayesinconsistent, at odds with updating based on a single model.

These findings challenge the descriptive validity of any updating rules that rely on aggregating the models into a single model. Hence, they not only question updating rules that aggregate models into a single model using the model prior as a Bayesian DM would do, but also those that aggregate competing models in a biased manner.

We can further explore the potential of more specific updating rules, in which the individual aggregates models using the model prior, but subsequently exhibits systematic biases when updating based on this single model, as discussed in Section D.3.1.<sup>36</sup> To do so, we study whether some participants consistently use some values of  $\alpha_s$  other than the ones associated with the updating rules already studied, i.e.,  $\alpha_s = 0$  (reporting the prior over the state) and  $\alpha_s = 1$  (Bayesian updating). Specifically, we consider the 289 participants who used both of these two rules in at most 4 tasks, as shown in Table 5. We then explore whether some of these individuals consistently use another  $\alpha_s$ . To account for some instability in  $\alpha_s$ , we consider three broad classifications:  $\alpha_s \in (1, \infty)$  (overinference about the state),  $\alpha_s \in [0, 1)$  (underinference about the states), and  $\alpha_s \in (-\infty, 0)$  (inference about the state in the wrong direction). Table D4 reports these results, indicating that a small share of participants use one of these three broad categories of updating rules consistently. This table also illustrates that updating in the wrong direction,  $\alpha_s \in (-\infty, 0)$ , is frequent. Hence, this class of updating rules fails to adequately capture our data.

This exercise also illustrates a potentially important insight for future research: investigating a setting with competing models while applying the standard approach—where individuals only entertain one model—can lead to incorrect conclusions. Using such an approach, we would conclude that participants' behavior is highly noisy—due to frequent updates in the wrong direction and Bayes-inconsistencies—and that, except for the 11 out of 300 participants who consistently use Bayes' rule or report the prior, updating patterns appear rather unstable. There is no consistent pattern of over- or underinference about the states as almost no participant systematically reports posteriors consistent with a stable parameter  $\alpha_s$ .

<sup>&</sup>lt;sup>36</sup>Our data is not well-suited to study biases in aggregating models into a compound model. Our approach here is to examine deviations from Bayesian updating by fixing the compound model. In principle, we could also do the opposite approach by assuming that participants use Bayes' rule to update their beliefs and then recovering the compound model they entertain—assuming that they indeed aggregate models into a single model. However, this alternative approach is not generally feasible. To apply it, we would also need (i) participants to report vectors of posteriors, and (ii) their vector of posterior beliefs to be Bayes-consistent. We have the reported vectors of posteriors for the repeated model pairs, of which 50% are Bayes-inconsistent. Thus, for half of the sample, there is no compound model that could explain the reported beliefs across signals. Note that this again highlights the difficulty of updating rules that assume participants aggregate models into a single model to explain our data.

Nr. Consistent	$c \in [0, 1)$	$\alpha \in (1, \infty)$	$\alpha \in (-\infty, 0)$
Observations	$\alpha_s \in [0, 1)$	$\alpha_s \in (1,\infty)$	$\alpha_s \in (-\infty, 0)$
0	31.14	2.42	19.03
1	38.06	7.96	14.88
2	16.96	19.38	15.22
3	8.30	25.95	23.53
4	4.50	24.57	14.19
5	1.04	12.80	9.69
6	0.00	6.23	3.11
7	0.00	0.69	0.35
Total	100.00	100.00	100.00

Table D4: Consistency of Updating Rules: Over- and Underinference about the State *Notes* The table shows how often different individuals use specific updating rules. Since participants complete 7 updating tasks, they can use each rule between 0 and 7 times (Column "Nr. Consistent Observations"). The columns then display the distribution of frequencies for different levels of  $\alpha_s$ . For example, Column " $\alpha_s \in [0, 1)$ " shows the share of participants who report guesses that correspond to any  $\alpha_s \in [0, 1)$  in 0, 1, 2, 3, 4, 5, 6, and 7 tasks.

### D.4 Use of Multiple Rules

In this section, we investigate whether some participants use multiple updating rules among the main rules studied so far. Among the 300 participants, 129 (43%) used one or multiple of the three updating rules—Bayesian updating, one-stage updating and/or model selection—in all 7 tasks (allowing for a distance of at most 2 p.p between the prediction and the reported guess). Additionally, 27 (9.00%) and 20 (6.67%) participants used one or multiple of these rules in 5 and 6 of the 7 updating tasks, respectively.<sup>37</sup> Tables D5,D6 and D7 present the distribution of the combinations of updating rules used by these different groups of participants.

In the following, we refer to a DM's updating pattern as XYZ, where X is the number of times the participant updates according to Bayes' rule (Column "# Bayes" in Table D5), Y denotes the number of times the participant follows the one-stage updating (Column "# One-stage"), and Z gives the number of time the participant selects one model (Column "# One model"). Note that a participant can be classified as using multiple rules in the same task when the predictions of these rules overlap. Consequently, X + Y + Z can exceed 7 (e.g., 170).

Table D5 shows that among participants who use one of the updating rules across all 7 updating tasks, a majority of 69.77% uses model selection in all 7 updating tasks (Row 007). Another 17.83% follow the one-stage updating rule in all 7 tasks (Rows 070 or 170), and 4.66% apply Bayes' rule across all 7 tasks (Rows 700 and 710). Only 7.75% of participants use different rules, with no particular combination of rules being notably frequent: 7 participants (5.43%) use a combination of one-stage updating and model

 $<sup>^{37}</sup>$ Furthermore, 25 (8.33%), 37 (12.33%), 29 (9.67%), 22 (7.33%), and 11 (3.67%) participants applied one or multiple of these rules for 4, 3, 2, 1, and 0 of the 7 tasks, respectively.

selection (Rows 016, 025, 043, 061, 134), 1 participant uses a combination of one-stage and Bayes' updating (Row 430), and 2 participants use a combination of all three rules (Rows 224, 261). Tables D6 and D7 also show no frequent or systematic combinations of updating rules among individuals who apply one rule for 6 or 5 out of the 7 tasks. Hence, we conclude that individuals rarely employ multiple rules.

# Bayesian	# One-stage	# Model Selection	# Obs.	Share
0	0	7	90	69.77
0	1	6	3	2.33
0	2	5	1	0.78
0	4	3	1	0.78
0	6	1	1	0.78
0	7	0	14	10.85
1	3	4	1	0.78
1	7	0	9	6.98
2	2	4	1	0.78
2	6	1	1	0.78
4	3	0	1	0.78
7	0	0	1	0.78
7	1	0	5	3.88
			129	100.00

Table D5: Using Different Updating Rules I

*Notes* The table focuses on the 129 participants who used either Bayes' updating, one-stage updating, or model selection, or a combination of these rules across all 7 updating tasks. We allow for a distance of at most 2 p.p between the prediction and the reported guess. The table then shows the distribution of rule combinations. For example, the first row reports the number (Column "# Obs.") and share (Column "Share") of participants who never used Bayes' rule (Column "# Bayes"), never used the one-stage rule (Column "# One-stage"), and 7 times selected one model (Column "# One model"). Note that Bayesian updating and one-stage updating can produce similar predictions in certain situations, so a single observation might be counted in both the "# Bayes" and "# One-stage" columns.
# Bayesian	# One-stage	# Model Selection	# Obs.	Share
0	0	6	15	55.56
0	1	5	3	11.11
0	2	4	1	3.70
0	3	3	1	3.70
0	6	0	3	11.11
1	0	5	1	3.70
1	6	0	2	7.41
2	0	4	1	3.70
			27	100.00

Table D6: Using Different Updating Rules

Notes The table focuses on the 27 participants who used either Bayes' updating, one-stage updating, or model selection, or a combination of these rules in 6 out of 7 updating tasks. We allow for a distance of at most 2 p.p between the prediction and the reported guess. The table then shows the distribution of rule combinations. For example, the first row reports the number (Column "# Obs.") and share (Column "Share") of participants who never used Bayes' rule (Column "# Bayes"), never used the one-stage rule (Column "# One-stage"), and s selected a model in six tasks (Column "# One model"). Note that Bayes' updating and one-stage updating can produce similar predictions in certain situations, so a single observation might be counted in both the "# Bayes" and "# One-stage" columns.

# Bayesian	# One-stage	# Model Selection	# Obs.	Share
0	0	5	6	30.00
0	1	4	1	5.00
0	2	3	1	5.00
0	5	0	1	5.00
1	0	4	1	5.00
1	1	4	1	5.00
1	2	2	1	5.00
1	4	1	1	5.00
1	5	0	1	5.00
2	1	2	1	5.00
2	2	2	1	5.00
2	3	0	1	5.00
3	3	0	1	5.00
3	3	1	1	5.00
5	1	0	1	5.00
			20	100.00

### Table D7: Using Different Updating Rules

Notes The table focuses on the 20 participants who used either Bayes' updating, one-stage updating, or model selection, or a combination of these rules across in 5 of the 7 updating tasks. We allow for a distance of at most 2 p.p between the prediction and the reported guess. The table then shows the distribution of rule combinations. For example, the first row reports the number (Column "# Obs.") and share (Column "Share") of participants who never used Bayes' rule (Column "# Bayes"), never applied the one-stage updating (Column "# One-stage"), and one selected one model in five tasks (Column "# One model"). Note that Bayesian updating and one-stage updating can produce similar predictions in certain situations, so a single observation might be counted in both the "# Bayes" and "# One-stage" columns.

# **E** Appendix: Bias and Response Times

In this section, we discuss deviations from the Bayesian benchmark and the response times associated with different updating rules.

First, we focus on deviations from the Bayesian benchmark, calculated as the absolute distance between the Bayesian prediction and the reported guess. This simple measure describes how different updating rules result in systematic bias in beliefs. Before looking at our data, we calculate the predicted average bias for each updating rule across all tasks. By definition, for a DM consistently applying Bayesian updating, the bias is 0. DMs who consistently use one-stage updating, select the best-fitting model, or select the worst-fitting models are predicted to have an average bias of 6.17 p.p., 15.7 p.p., and 28.24 p.p., respectively. If a DM applies a stochastic version of model selection via maximum likelihood, where mistakes result in selecting the worst-fitting model 8.83% of the time (as estimated in Appendix Section C), the average bias is predicted to be 16.81 p.p..<sup>38</sup>

In Figure E1, we present the empirical distances from the Bayesian benchmark for individuals consistently using Bayesian updating, one-stage updating, or model selection. The figure shows the average absolute distance between guesses and the Bayesian benchmark across all seven updating tasks for the 158 participants who used these rules at least 5 out of 7 times. Consistent with the predicted biases implied by these rules, the average distance is 2.04 p.p. for participants classified as using Bayesian updating, 6.63 p.p. for those using one-stage updating, and 20.52 p.p. for those using model selection.

Figure E2 examines the 90 participants who applied model selection in all seven updating tasks, categorizing them by how often they selected the best-fitting model. The average bias increases monotonically, from 28.38 p.p. for those who never selected the best-fitting model to 16.64 p.p. for those who always did.

This analysis highlights that model selection results in substantially greater deviations from the Bayesian benchmark than one-stage updating, even for participants consistently selecting the best-fitting model (which minimizes the distance to the Bayesian benchmark conditional on using model selection).

These deviations may be acceptable for some participants if they lead to substantial time savings, meaning updating posterior beliefs more quickly. Figure E3 shows the median response times across all seven tasks for individuals consistently using different updating rules. The figure illustrates that both one-stage updating and model selection are associated with substantially shorter response times compared to Bayesian updating: the median response time is 74.62 seconds for participants classified as following Bayesian updating, 19.18 seconds for participants classified as following one-stage updating, and 17.96 seconds for those classified as following model selection. This indicates that DMs face a trade-off between precision and time.

 $<sup>^{38}\</sup>mathrm{The}$  predicted average distance is 21.66 p.p. using model selection based on informativeness.



Figure E1: Deviations from the Bayesian Benchmark for Different Updating Rules

*Notes.* The figure presents a box plot of the average absolute distance between the reported guesses and the Bayesian prediction in all 7 tasks for participants who consistently used one updating rule. The sample consists of the 158 participants that consistently used the Bayesian updating ("Bayesian"), one-stage updating ("One-stage"), or model selection ("Model Selection") in at least 5 out of 7 times, allowing for a distance of at most 2 p.p between the prediction and the reported guess (see Table 5). Due to the small number of observations, we do not include participants who consistently reported the prior.



Figure E2: Deviations from the Bayesian Benchmark for Model Selection

*Notes.* The figure presents a box plot of the average absolute distance between the reported guesses and the Bayesian prediction in all tasks for participants who consistently used model selection. We categorize individuals based on how often they select the best-fitting model (ranging from 0 to 7 times). The sample consists of the 90 participants that consistently select a model in all 7 updating tasks, allowing for a distance of at most 2 p.p between the prediction and the reported guess (see Table 5).



Figure E3: Response Times for Different Updating Rules

*Notes.* The figure presents a box plot of the median response time in all tasks for participants who consistently used different updating rules. The sample consists of the 158 participants that consistently used the Bayesian ("Bayesian"), one-stage ("One-stage"), or model selection ("Model Selection") updating rules in at least 5 out of 7 times, allowing for a distance of at most 2 p.p between the prediction and the reported guess (see Table 5). Due to the small number of observations, we do not include participants who consistently reported the prior.

# F Appendix: Additional Data Collection

This section includes additional information on our second study. First, we describe the details of the experimental design and data collection. Then, we include additional results and figures using this data. We discuss these results in Section 4.7 of the main paper.

# F.1 Experimental Design

The focus of this second study is to test the generalizability of our results in other settings as motivated in Section 4.7. To do so, we closely replicate the design presented in Section 3, including instructions, with a few modifications that we discuss in this section.

First, we shorten the experiment by reducing the number of updating tasks in Part 3 from seven to two, using Model Pair 2 and Model Pair 3. These pairs are well-suited for identifying updating rules, as the predictions of different rules are particularly distinct (see Appendix Table B1). This is also why we used them as the repeated model pairs in the main study. Participants encounter these two model pairs in random order.

Second, we change how we present the selection of the implemented model to participants. In our main study, we explained to participants that a die roll would randomly determine which model was implemented in that task: if the number 1, 2, or 3 is rolled, Model 1 was selected, while otherwise Model 2 was selected. To avoid restricting possible model priors to a small set in the *No-Prior* condition, we replace the die roll with a card draw in all treatments. The card is drawn from a deck of 100 red and blue cards, where red cards correspond to Model 1 (a red robot) and blue cards correspond to Model 2 (a blue robot). In the *Baseline* and *Click* conditions, participants know that 50 cards are red and 50 cards are blue, implementing the same objective prior over models as in our main study. Instead, in the *No-Prior* condition, participants do not know how many cards are blue or red. A control question verifies the comprehension of this aspect.

Third, since this study required a shorter completion time and half the tasks of our main study, we adjusted the completion fee to 5.7 USD and the bonus payment to 1 USD.

Fourth, we shorten Part 4, where participants fill out a short survey that only includes a modified version of the Cognitive Reflection Test (Frederick, 2005) and demographics.

The only difference between the *No-Prior* and *Baseline* conditions lies in whether the color composition of the cards is known to participants. The *Click* condition differs from *Baseline* in that model predictions are not automatically displayed to participants. Instead, participants can access each prediction, if they want, by clicking a corresponding button five times in a row. This feature is introduced in Part 2 of the study, at the same time as the predictions. In Part 2, participants must press the "G" button to reveal the prediction of the model (represented by a green robot). In Part 3, they must press the "R" button to reveal the prediction of the first model (red robot) and the "B" button for the second model (blue robot). Participants cannot reveal both predictions simultaneously.

**Logistics** The experiment was pre-registered on AsPredicted and conducted on Prolific in February 2025, restricting the participant pool to US residents, aged 18-70, with approval rates of at least 95%.<sup>39</sup> The study was completed through a link to a Qualtrics survey, including instructions and control questions for each part (see Appendix G). A total of 592 participants completed the study successfully. Of these participants, 34% and 37% were randomly allocated to *Baseline* and *No-Prior* conditions, respectively. This difference in treatment allocation is purely random: participants can only be excluded for failing attention checks, which take place at the beginning of the study when all treatments are the same. The average payment was 6.5 USD, and the average duration was approximately 30 minutes. In our final sample, 49% are female, 24% have low schooling ('High school' or lower educational level), and the median age is 37.

## F.2 Additional Results





Notes. Cumulative distribution of weights on the best-fitting model,  $\rho$ , across tasks for each treatment condition. We report the implied weights for the reported guesses within the two model predictions.



Figure F2: Cumulative Distribution of  $\alpha$ 

Notes. Cumulative distribution of  $\alpha$  across tasks for each treatment condition (D.1). We exclude observations for which reported guesses fall outside the model predictions. Note that almost all observations with  $\alpha < -10$  and  $\alpha^m > 10$  correspond to selecting the worst- or best-fitting model, respectively.

<sup>&</sup>lt;sup>39</sup>The pre-registration plan is available at https://aspredicted.org/wqtd-68zm.pdf.

Type of Guess	Baseline		]	No-Prior	Click		
	%	95%-CI	%	95%-CI	%	95%-CI	
Bayesian	0.75	[0.00, 1.59]	0.91	[0.00, 2.01]	2.03	[0.54, 3.53]	
One-stage	8.71	[5.13, 12.28]	8.90	[5.41, 12.40]	11.92	[7.57, 16.27]	
Best-fitting	36.07	[30.28, 41.85]	36.99	[31.61, 42.36]	38.95	[32.40, 45.51]	
Worst-fitting	10.70	[7.37, 14.02]	7.76	[5.10, 10.42]	9.88	[6.19, 13.58]	
Within	31.09	[25.55, 36.64]	29.91	[24.75, 35.06]	25.29	[19.68, 30.90]	
Outside	12.69	[8.81, 16.56]	15.53	[11.89, 19.16]	11.92	[8.22, 15.61]	
Total	100.00		100.00		100.00		

Table F1: Classification of Guesses by Treatment, Exact Match (Second Study)

*Notes.* The table reports the shares of guesses that exactly match each point prediction (Bayesian, one-stage, best-fitting, and worst-fitting) for each treatment condition (Baseline, No-Prior, Click). Columns "%" and "95%-CI" report the shares and corresponding 95% confidence intervals, using standard errors clustered at the individual level. We pool the data from both updating tasks.

Type of Guess		Exact	Within 2 p.p.			
	%	95%-CI	%	95%-CI		
Bayesian	3.17	[1.43, 4.90]	5.50	[3.34, 7.66]		
One-stage	9.17	[6.25, 12.09]	15.33	[11.68, 18.99]		
Best-fitting	32.67	[28.02, 37.31]	38.33	[33.60, 43.07]		
Worst-fitting	7.50	[5.17, 9.83]	9.67	[7.11, 12.23]		
Within	33.00	[28.46, 37.54]				
Outside	14.50	[11.39, 17.61]				
Total	100.00		68.83			

Table F2: Classification of Guesses by Treatment (Main Study)

Notes. Column "Exact" reports the shares of guesses that can be exactly classified as one of the point predictions (Bayesian, one-stage, best-fitting, and worst-fitting) or in one of the two residual categories ("Within" if the guess is within the model predictions or "Outside" otherwise). Column "Within 2 p.p." reports the shares of guesses that fall within 2 p.p. around each point prediction. We pool the data from both Model Pair 2 and Model Pair 3 before the repetitions from the main study (N=600).

Type of Guess	Baseline			No-Prior	Click		
	%	95%-CI	%	95%-CI	%	95%-CI	
Bayesian	3.98	[1.25, 6.71]	7.76	[4.19, 11.33]	5.81	[2.28, 9.35]	
One-stage	13.93	[9.10, 18.76]	16.44	[11.49, 21.39]	17.44	[11.71, 23.17]	
Best-fitting	41.29	[34.43, 48.16]	43.38	[36.76, 49.99]	40.70	[33.28, 48.11]	
Worst-fitting	13.93	[9.10, 18.76]	10.96	[6.79, 15.13]	11.63	[6.79, 16.47]	
Total	73.13		78.54		75.58		

Table F3: Classification of Guesses by Treatment, Model Pair 2 (Second Study)

*Notes.* The table reports the shares of guesses that fall within 2 p.p. around each point prediction (Bayesian, one-stage, best-fitting, and worst-fitting) for each treatment condition (Baseline, No-Prior, Click). Columns "%" and "95%-CI" report the shares and corresponding 95% confidence intervals, using standard errors clustered at the individual level. We only use the data from Model Pair 2.

Type of Guess	Baseline			No-Prior	Click		
	%	95%-CI	%	95%-CI	%	95%-CI	
Bayesian	3.48	[0.93, 6.04]	1.37	[0.00, 2.92]	4.07	[1.09, 7.05]	
One-stage	10.45	[6.18, 14.71]	10.50	[6.41, 14.59]	14.53	[9.21, 19.86]	
Best-fitting	46.77	[39.81, 53.72]	43.84	[37.21, 50.46]	48.26	[40.71, 55.80]	
Worst-fitting	8.96	[4.97, 12.94]	9.13	[5.29, 12.98]	11.05	[6.31, 15.78]	
Total	69.66		64.84		77.91		

Table F4: Classification of Guesses by Treatment, Model Pair 3 (Second Study)

*Notes.* The table reports the shares of guesses that fall within 2 p.p. around each point prediction (Bayesian, one-stage, best-fitting, and worst-fitting) for each treatment condition (Baseline, No-Prior, Click). Columns "%" and "95%-CI" report the shares and corresponding 95% confidence intervals, using standard errors clustered at the individual level. We only use the data from Model Pair 3.

Type of Guess		Baselin	e vs. 1	No-Prior	Baseline vs. Click			
	Coeff.	$\mathbf{t}$	р	95%-CI	Coeff.	$\mathbf{t}$	р	95%-CI
Bayesian	0.81	0.57	0.57	[-1.99, 3.60]	1.30	0.85	0.40	[-1.71, 4.31]
One-stage	1.02	0.35	0.73	[-4.67, 6.71]	3.80	1.19	0.24	[-2.49, 10.09]
Best-fitting	-0.53	-0.13	0.90	[-8.74, 7.68]	0.31	0.07	0.95	[-8.64, 9.26]
Worst-fitting	-1.24	-0.53	0.59	[-5.80, 3.32]	-0.08	-0.03	0.98	[-5.21, 5.06]
Model Selection	-1.77	-0.40	0.69	[-10.49,  6.94]	0.24	0.05	0.96	[-9.31, 9.79]
Best-fitting, cond.	1.69	0.43	0.67	[-6.08, 9.47]	0.13	0.03	0.98	[-8.52, 8.77]
Identified	-0.69	-0.18	0.86	[-8.17, 6.78]	4.96	1.26	0.21	[-2.78, 12.71]

Table F5: Treatment Comparisons (Second Study)

Notes. This table presents treatment effects by regressing indicator variables, capturing whether participants are classified under different updating rules (allowing for a 2 p.p.; see Table 8), on treatment dummies. The analysis pools data from both updating tasks, clusters standard errors at the individual level, and includes model pair x signal fixed effects. The column "Baseline vs. No-Prior" compares the Baseline and No-prior conditions (N=840), while the column "Baseline vs. Click" compares the Baseline and Click conditions (N=746). For the row "Model Selection," the outcome variable is defined as 1 if the participant selects either the best- or worst-fitting model and 0 otherwise. For the row "Best-fitting, cond.," we restrict the sample to observations where participants engage in model selection (N=458 for No-prior and N=415 for Click). For the row "Identified," the outcome variable is defined as 1 if the observation is classified as Bayesian, One-stage, Best-fitting or Worst-fitting, and 0 otherwise.

Type of Guess	Baseline vs. No-Prior					Baseline vs. Click			
	Coeff.	$\mathbf{t}$	р	95%-CI	Coeff.	$\mathbf{t}$	р	95%-CI	
Bayesian	0.17	0.23	0.82	[-1.24, 1.57]	1.29	1.46	0.14	[-0.45, 3.03]	
One-stage	0.01	0.01	1.00	[-4.95, 4.97]	3.16	1.11	0.27	[-2.44, 8.76]	
Best-fitting	0.88	0.22	0.83	[-7.05, 8.80]	2.66	0.60	0.55	[-6.03, 11.35]	
Worst-fitting	-2.76	-1.29	0.20	[-6.97, 1.45]	-0.80	-0.32	0.75	[-5.76, 4.16]	
Model Selection	-1.88	-0.43	0.67	[-10.53, 6.76]	1.86	0.39	0.70	[-7.59, 11.32]	
Best-fitting, cond.	5.35	1.25	0.21	[-3.05, 13.74]	2.34	0.48	0.63	[-7.20, 11.87]	
Identified	-1.71	-0.40	0.69	[-10.12, 6.71]	6.32	1.40	0.16	[-2.56, 15.20]	

Table F6: Treatment Comparisons, Exact Match (Second Study)

Notes. This table presents treatment effects by regressing indicator variables, capturing whether participants are classified under different updating rules (requiring an exact match; see Table F1), on treatment dummies. The analysis pools data from both updating tasks, clusters standard errors at the individual level, and includes model pair x signal fixed effects. The column "Baseline vs. No-Prior" compares the Baseline and No-prior conditions (N=840), while the column "Baseline vs. Click" compares the Baseline and Click conditions (N=746). For the row "Model Selection," the outcome variable is defined as 1 if the participant selects either the best- or worst-fitting model and 0 otherwise. For the row "Best-fitting, cond.," we restrict the sample to observations where participants engage in model selection (N=458 for No-prior and N=415 for Click). For the row "Identified," the outcome variable is defined as 1 if the observation is classified as Bayesian, One-stage, Best-fitting or Worst-fitting, and 0 otherwise.

	Bayesian	One-stage	Best-fitting	Worst-fitting	Other
Bayesian	0.34	0.68	0.34	0.00	2.70
One-stage	0.68	8.45	2.36	0.34	4.05
Best-fitting	0.34	0.00	31.59	4.73	5.24
Worst-fitting	0.00	0.17	7.43	3.21	1.35
Other	1.52	2.36	4.39	1.35	16.39

Table F7: Consistency of Updating Rules (Second Study)

*Notes* This table reports the correlation between the classifications of the updating rules in Model Pair 2 (rows) and Model Pair 3 (columns) within participants. We allow for a distance of at most 2 p.p between the prediction and the reported guess.

Dependent Variable		Sel	ects Mode	l 1	
	(1)	(2)	(3)	(4)	(5)
Best-fitting	0.602***		$0.595^{***}$		$0.595^{***}$
	(0.034)		(0.035)		(0.035)
Most Informative		-0.141***	-0.044		
		(0.038)	(0.032)		
Model Pair 2				0.141***	0.044
				(0.038)	(0.032)
Constant	0.192***	$0.547^{***}$	0.218***	0.406***	0.173***
	(0.022)	(0.028)	(0.031)	(0.027)	(0.024)
Observations	650	650	650	650	650
$R^2$	0.362	0.020	0.364	0.020	0.364

Table F8: Criteria for Model Selection (Second Study)

Notes. The table shows coefficient estimates from linear regressions of an indicator of selecting Model 1 on different explanatory variables. The sample consists of the 650 guesses that correspond to model selection (allowing for a 2 percentage point margin; see Table 8). "Best-fitting" is an indicator for Model 1 having a higher fit than Model 2 given the observed signal. "Most informative" is an indicator for Model 1 being more informative about the state than Model 2 given the observed signal. "Model Pair 2" is a model pair fixed effects that capture any factors that are unconditional on the signal, which, in this second study, also includes informativeness. Standard errors are clustered on the individual level (372 clusters) and are shown in parentheses. \* p < 0.10, \*\* p < 0.05, \*\*\* p < 0.01

Dependent Variable		Sel	ects Mode	l 1	
	(1)	(2)	(3)	(4)	(5)
Best-fitting	$0.597^{***}$		$0.590^{***}$		0.590***
	(0.037)		(0.038)		(0.038)
Most Informative		-0.142***	-0.040		
		(0.042)	(0.035)		
Model Pair 2				0.142***	0.040
				(0.042)	(0.035)
Constant	0.197***	0.540***	0.221***	0.397***	0.181***
	(0.024)	(0.031)	(0.034)	(0.029)	(0.026)
Observations	552	552	552	552	552
$R^2$	0.354	0.020	0.356	0.020	0.356

#### Table F9: Criteria for Model Selection, Exact match (Second Study)

Notes. The table shows coefficient estimates from linear regressions of an indicator of selecting Model 1 on different explanatory variables. The sample consists of the 552 guesses that correspond to model selection (requiring an exact match; see Table F1). "Best-fitting" is an indicator for Model 1 having a higher fit than Model 2 given the observed signal. "Most informative" is an indicator for Model 1 being more informative about the state than Model 2 given the observed signal. "Most informative" is an indicator for Model 1 being fixed effect that capture any factors that are unconditional on the signal, which, in this second study, also includes informativeness. Standard errors are clustered on the individual level (329 clusters) and are shown in parentheses. \* p < 0.10, \*\*\* p < 0.05, \*\*\* p < 0.01

# G Appendix: Experimental Instructions & Interface

## G.1 Main Study

#### OVERVIEW

The study takes about 30 minutes to complete. You are guaranteed to receive \$6 for completing the study.

This study consists of 4 parts. In the **first 3 parts**, you will be asked to complete several tasks in a similar format. This format is based on a **guessing task**. You can earn an additional bonus of **\$2 by making accurate guesses**. You will receive more detailed instructions about the guessing task and the bonus for accuracy before you participate in each part. *In these 3 parts, you will complete a total of 11 guessing tasks*. You will be paid the bonus payment for 1 of these 11 tasks, chosen at random by the computer. This means that you should consider the decisions you make in each part carefully. In the fourth part, you will fill out a questionnaire.

Given the instructions above, what is the guaranteed dollar payment that you receive for completing the study? (Please make a numeric entry)

Given the instructions above, about how many minutes does it take to complete the study? (Please make a numeric entry)

**--**

#### **INSTRUCTION - PART 1**

In Part 1, you are asked to complete 2 guessing tasks. In each guessing task, there are 2 bags, bag A and bag B. Each bag contains 6 balls, some of which are orange and some of which are purple. A robot chooses the number of orange and purple balls in each bag. For example, robot L7 chose the color of the balls in the bags as shown in the following figure:



In each guessing task, the computer flips a **fair coin** to select bag A or bag B. It is **equally likely** that the computer selects bag A or bag B. You will not observe which bag was **selected.** Instead, the computer draws one ball from the secretly selected bag and shows you the color of the ball.

The goal of the guessing task is to correctly guess the chance (in %) that bag A was selected, given the available information. You make your guess by selecting the chance between 0% and 100% on a slider. To give some examples of what these percentages mean:

- · 50% means that you think it is equally likely that bag A and bag B were selected,
- 0% means that you are certain that bag B was selected,
- · 100% means that you are certain that bag A was selected.

Remember that you can earn an additional \$2 by making accurate guesses. We have carefully chosen the payment rule such that you maximize the chance of winning the bonus of \$2 if you give your best guesses. It is in your best interest to always give the guess that you think is the true chance. The closer your guess is to the true chance, the higher the probability of winning the bonus. If you are interested, further details on the payment are provided <u>here</u>.

In Part 1, you will complete 2 different guessing tasks. In each task, a different robot chooses the number of orange and purple balls in each bag. The computer then randomly selects (1) a bag, and (2) a ball from the selected bag. Which bag is selected in each task is independent of the selected bag in other tasks.

Before you report your guesses, please answer the following comprehension questions.

1. What does the robot do in the guessing task? (Select all options that are correct)

a) Choose the number of orange and purple balls in each bag
b) Flip a fair coin to select bag A or bag B
c) Randomly pick a ball from the selected bag

2. What does the computer do in the guessing task? (Select all options that are correct)

a) Choose the number of orange and purple balls in each bag

b) Flip a fair coin to select bag A or bag B

c) Randomly draw a ball from the selected bag

3. What do you know when you guess the chance that bag A was selected after observing the drawn ball? (Select all options that are correct)

a) The number of orange and purple balls in each bag

b) Whether bag A or bag B was selected

c) The color of the ball that is randomly drawn from the selected bag

4. What can you earn by making accurate guesses?

a) Nothing	
b) Up to \$0.2	
c) Up to \$2	

5. Is the following statement true or false? "It is in your best interest to give your best guess of the chance that bag A was selected given the color of the drawn ball."

a) True			
b) False			

--->

#### PART 1: Task 1/2

Please click on the right arrow if you are ready to proceed to the next task.

For this task, consider robot L7.



When you click "START', the following steps take place:

- Step 1: the computer fills the bags with purple and orange balls according to robot L7
- Step 2: the computer selects one bag by flipping a fair coin
- Step 3: the computer draws a ball from the bag that was selected in Step 2

START

Step 1: The computer filled the bags with balls according to robot L7.



Step 2: The computer flipped the coin and has selected the bag.



Step 3: The computer **has drawn a random ball** from the selected bag in Step 2. **The drawn ball is orange.** 



Remember that for this task we consider robot L7.



#### Remember, the computer has drawn an orange ball

What do you think is the chance that bag A was selected given that the color of the drawn ball is orange? Make your guess using the slider

Chance of bag A (in %): Please click on the slider

#### INSTRUCTION - PART 2

Part 2 is designed to help you understand an **important aspect of the guessing task that will be important for Part 3**: There is a **mathematical formula** (called the Bayes rule) that gives the **correct chance** that bag A was selected by the computer if you observe a ball of a certain color.

For example, consider again robot L7 that chose the color of the balls in the bags as shown in the following figure:



According to this formula:

- · If the observed ball is orange, the correct chance that bag A was selected is 17%,
- . If the observed ball is purple, the correct chance that bag A was selected is 83%.

In Part 2, you will complete the same guessing tasks as in Part 1, but now the robots inform you of the correct chance that bag A was selected given the color of the drawn ball. Note that the robots always communicate the correct chance; that is, the robots never lie to you and never make mistakes when calculating the correct chance.

The following figure provides an example of this interface:



Again you make your guess by selecting the chance between 0% and 100% on a slider. Now, a colorful pin helps you to visually indicate on the slider the correct chance communicated by the robot (see example above).

Remember that the closer your guess is to the correct chance, the higher the chance of winning the bonus of \$2. That is, **you maximize the chance of winning the bonus by submitting the correct guess provided by the robot**. In the example above, if a purple ball was drawn, you would maximize the chance of winning the bonus by reporting a guess of 83%.

The goal of Part 2 is to familiarize with the interface where the robots communicate the correct chances. Understanding the interface is important for Part 3, as we will explain later.

1) Is the following statement true or false? "The robots never lie to you and never make mistakes when calculating the correct chance that bag A was selected."

a) True			
b) False			

#### 2) Consider the following example:

Bag A has 1 purple ball and 5 orange balls • • • • • •	Bag B has 5 purple balls and 1 orange ball • • •	
Remem	ber, the comput	er has drawn a purple ball
The chance that bag A was selected given that the color of the drawn ball is purple would be 17%		

a) What is the correct chance (in %) that bag A was selected given that the color of the drawn ball is purple?

L	4	-		
L		1		

b) What guess (in %) would maximize your chance of winning the bonus?

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#### PART 2: Task 1/2

Please click on the right arrow if you are ready to proceed to the next task.

For this task, consider robot L7.



When you click "START', the following steps take place:

- Step 1: the computer fills the bags with purple and orange balls according to robot L7
- Step 2: the computer selects one bag by flipping a fair coin
- Step 3: the computer draws a ball from the bag that was selected in Step 2

START

Step 1: The computer filled the bags with balls according to robot L7.



Step 2: The computer flipped the coin and has selected the bag.



Step 3: The computer **has drawn a random ball** from the selected bag in Step 2. **The drawn ball is orange.** 



Remember that for this task we consider robot L7.



#### Remember, the computer has drawn an orange ball

The chance that bag A was selected given that the color of the drawn ball is orange would be 17%

What do you think is the chance that bag A was selected given that the color of the drawn ball is orange? Make your guess using the slider

Chance of bag A (in %): Please click on the slider

#### INSTRUCTION - PART 3

You will now complete a **new version of the guessing task**. Some elements are the same as the previous version of the guessing task: there are again 2 bags, bag A and bag B; each bag contains 6 balls, and each ball is either orange or purple. Now, in each task, there are 2 robots. Each robot suggests a distribution of purple and orange balls in the bags. In the following, we illustrate the new guessing tasks with the robots H8 and Z5. Note, however, that we will use two different robots in each task. The following figure shows you how the robots H8 and Z5 would distribute the balls in the bags for this task:



The suggestion of one of the two robots is used to fill the bags with the balls. First, the computer randomly selects one of the two robots, either robot H8 or robot Z5, and then fills bag A and bag B according to how the selected robot would distribute the balls in the bags. The suggestion of the other robot does not count. To select a robot, the computer rolls a fair 6-sided die.

- If the number 1, 2 or 3 was rolled, the computer chooses robot H8.
- If the number 4, 5 or 6 was rolled, the computer chooses robot Z5.

Importantly, you do not know what number was rolled and whether robot H8 or robot Z5 was selected by the computer, and, hence, you do not know the composition of bag A and bag B.

After the computer selects a robot and fills the bags according to this robot's suggestion, the computer then flips a fair coin to select bag A or bag B as in the guessing tasks that you did in Part 1 and Part 2. It is equally likely that the computer selects bag A or bag B. As before, you will not observe which bag was selected. Instead, the computer draws one ball from the secretly selected bag and shows you the color of the ball.

As in Part 2, the robots tell you the correct chance that the observed ball was drawn from bag A, but each robot tells you the correct chance for the case in which the robot's suggestion was selected to fill bag A and bag B. In the example of robots H8 and Z5, consider the case in which an orange ball was drawn. In this case:

- · If the robot H8's suggestion were selected, the correct chance would be 63%,
- If the robot Z5's suggestion were selected, the correct chance would be 33%.

Your goal of the new guessing task is to correctly guess the chance (in %) that bag A was selected, given the available information. Again, you make your guess by selecting the chance between 0% and 100% on a slider. Two colorful pins help you to visually indicate on the slider the correct chance if if the suggestion of robot H8 is selected (63%) and the correct chance if the suggestion of robot Z5 is selected (33%):



As in the previous parts, to maximize the chance of winning the bonus of \$2, it is in your best interest to always give a guess that you think is the true chance.

In Part 3, you will complete 7 of these new guessing tasks. Note that draws in these guessing tasks are independent of any earlier draws. So you should think about which bag was selected for each task independently of all other tasks.

Before you report your guesses, please answer the following comprehension questions.

1. What does the computer do in the new guessing task? (Select all options that are correct)

a) Choose the number of orange and purple balls in each bag for both robots

b) Roll a fair 6-sided die to select a robot, and uses the chosen numbers of this robot to fill bag A and bag B

c) Flip a fair coin to select bag A or bag B

d) Randomly draw a ball from the selected bag of the selected robot

2. Consider the following example:



Remember, the computer has drawn a purple ball.

The chance that bag A was selected given that the color of the drawn ball is purple would be...

71% if I, robot X1, were selected 17% if I, robot H5, were selected

a) Suppose that the computer randomly chose robot X1 to fill bag A and bag B. What is the correct chance (in %) that the bag A was selected given that the color of the drawn ball is purple?

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b) Suppose that the computer randomly chose robot H5 to fill bag A and bag B. What is the correct chance (in %) that the bag A was selected given that the color of the drawn ball is purple?

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c) What do you know when you guess the chance that bag A was selected? (Select all options that are correct)



d) The color of the ball that is randomly drawn from the selected bag

#### PART 3: Task 1/7

Please click on the right arrow if you are ready to proceed to the next task.

#### For this task, consider robot P3 and robot V6.



When you click "START', the following steps take place:

- · Step 1: the computer selects one of the two robots by rolling a fair 6-sided die
  - If the number 1, 2 or 3 is rolled: the computer selects robot P3
  - If the number 4, 5 or 6 is rolled: the computer selects robot V6
- Step 2: the computer fills the bags with purple and orange balls according to the robot selected in Step 1
- · Step 3: the computer selects one bag by flipping a fair coin
- · Step 4: the computer draws a ball from the bag that was selected in Step 3

#### START

Step 1: The computer rolled the dice and has selected the robot.



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Step 2: The computer filled the bags with balls according to the selected robot in Step 1.



Step 3: The computer flipped the coin and has selected the bag.



Step 4: The computer **has drawn a random ball** from the selected bag in Step 2. **The drawn ball is purple.** 



Remember that for this task we consider robot P3 and robot V6.



Remember, the computer has drawn a purple ball

The chance that bag A was selected given that the color of the drawn ball is purple would be...

86% if I, robot P3, were selected

33% if I, robot V6, were selected

What do you think is the chance that bag A was selected given that the color of the drawn ball is purple? Make your guess using the slider

Chance of bag A (in %): Please click on the slider

## G.2 Second Study

### G.2.1 Part 1: All Treatments

#### WELCOME!

This study on decision-making is conducted by researchers at the **University of Copenhagen, Denmark** and **Pompeu Fabra University, Spain.** 

It is important that you take the time to read all instructions and that you read questions carefully before you answer them.

The study takes about **30 minutes** to complete. You will receive a fixed payment of **\$5.7** (£4.5) for completing the study.

Your participation in this study does not pose any risks to your health or well-being. All the information gathered in this study is kept strictly anonymous and will never be associated with your identity.

By continuing you acknowledge that you understand the above information and give your consent to participate in the study. Participation in this study is voluntary. You may withdraw your consent and discontinue participation in the project at any time. Doing so will not result in any penalty.

If you choose not to participate, please click "I do NOT want to participate" at the bottom of this page. If you want to participate, please choose "I want to participate."

I want to participate

I do NOT want to participate



#### OVERVIEW

The study takes about 30 minutes to complete. You are guaranteed to receive 5.7 (£4.5) for completing the study.

This study consists of 4 parts. In the **first 3 parts**, you will be asked to complete several tasks in a similar format. This format is based on a **guessing task**. You can earn an additional bonus of **\$1 by making accurate guesses**. You will receive more detailed instructions about the guessing task and the bonus for accuracy before you participate in each part. *In these 3 parts, you will complete a total of 6 guessing tasks*. You will be paid the bonus payment for 1 of these 6 tasks, chosen at random by the computer. This means that you should consider the decisions you make in each part carefully. In the fourth part, you will fill out a short questionnaire.

Given the instructions above, about how many minutes does it take to complete the study? (Please make a numeric entry)

**→** 

#### **INSTRUCTION – PART 1**

In Part 1, you are asked to complete 2 guessing tasks. In each guessing task, there are 2 bags, bag A and bag B. Each bag contains 6 balls, some of which are orange and some of which are purple. A robot chooses the number of orange and purple balls in each bag. For example, robot L7 chose the color of the balls in the bags as shown in the following figure:



In each guessing task, the computer flips a **fair coin** to select bag A or bag B. It is **equally likely** that the computer selects bag A or bag B. **You will not observe which bag was selected.** Instead, the computer draws one ball from the secretly selected bag and shows you the color of the ball.

The goal of the guessing task is to correctly guess the chance (in %) that bag A was selected, given the available information. You make your guess by selecting the chance between 0% and 100% on a slider. To give some examples of what these percentages mean:

- 50% means that you think it is equally likely that bag A and bag B were selected,
- 0% means that you are certain that bag B was selected,
- 100% means that you are certain that bag A was selected.

Remember that you can earn an additional \$1 by making accurate guesses. We have carefully chosen the payment rule such that you maximize the chance of winning the bonus of \$1 if you give your best guesses. It is in your best interest to always give the guess that you think is the true chance. The closer your guess is to the true chance, the higher the probability of winning the bonus. If you are interested, further details on the payment are provided <u>here</u>.

In Part 1, you will complete 2 different guessing tasks. In each task, a different robot chooses the number of orange and purple balls in each bag. The computer then randomly selects (1) a bag, and (2) a ball from the selected bag. Which bag is selected in each task is independent of the selected bag in other tasks.

Before you report your guesses, please answer the following comprehension questions.

1. What does the robot do in the guessing task? (Select all options that are correct)

a) Choose the number of orange and purple balls in each bag

b) Flip a fair coin to select bag A or bag B

c) Randomly pick a ball from the selected bag

2. What does the computer do in the guessing task? (Select all options that are correct)

a) Choose the number of orange and purple balls in each bag

b) Flip a fair coin to select bag A or bag B

c) Randomly draw a ball from the selected bag

3. What do you know when you guess the chance that bag A was selected after observing the drawn ball? (Select all options that are correct)

a) The number of orange and purple balls in each bag

b) Whether bag A or bag B was selected

c) The color of the ball that is randomly drawn from the selected bag

5. Is the following statement true or false? "It is in your best interest to give your best guess of the chance that bag A was selected given the color of the drawn ball."

a) True

b) False

PART 1: Task 1/2

Please click on the right arrow if you are ready to proceed to the next task.

 $\rightarrow$ 

For this task, consider robot L7.



When you click "START", the following steps take place:

- Step 1: the computer fills the bags with purple and orange balls according to robot L7
- Step 2: the computer selects one bag by flipping a fair coin
- Step 3: the computer draws a ball from the bag that was selected in Step 2

START

Step 1: The computer filled the bags with balls according to robot L7.



Step 2: The computer flipped the coin and has selected the bag.



Step 3: The computer **has drawn a random ball** from the selected bag in Step 2. **The drawn ball is orange.** 



Remember that for this task we consider robot L7.



#### Remember, the computer has drawn an orange ball

What do you think is the chance that bag A was selected given that the color of the drawn ball is orange? Make your guess using the slider

Chance of bag A (in %): Please click on the slider

### G.2.2 Part 2: Baseline, No-Prior

#### **INSTRUCTION - PART 2**

Part 2 is designed to help you understand an **important aspect of the guessing task that will be important for Part 3**: There is a **mathematical formula** (called the Bayes rule) that gives the **correct chance** that bag A was selected by the computer if you observe a ball of a certain color.

For example, consider again robot L7 that chose the color of the balls in the bags as shown in the following figure:



According to this formula:

- If the observed ball is orange, the correct chance that bag A was selected is 17%,
- If the observed ball is purple, the correct chance that bag A was selected is 83%.

In Part 2, you will complete the same guessing tasks as in Part 1, but **now the robots** inform you of the correct chance that bag A was selected given the color of the drawn ball. Note that the robots always communicate the correct chance; that is, the robots never lie to you and never make mistakes when calculating the correct chance.

Bag A has 5 purple balls and 1 orange ball ••••
Remember, the computer has drawn a purple ball
The chance that bag A was selected given that the color of the drawn ball is purple is
83%
What do you think is the chance that bag A was selected given that the color of the drawn ball is purple? Make your guess using the slider
Chance of bag A (in %): Please click on the slider

The following figure provides an example of the interface:

Again you make your guess by selecting the chance between 0% and 100% on a slider. Now, a colorful pin helps you to visually indicate on the slider the correct chance communicated by the robot (see example above).

Remember that the closer your guess is to the correct chance, the higher the chance of winning the bonus of \$1. That is, **you maximize the chance of winning the bonus by submitting the correct guess provided by the robot**. In the example above, if a purple ball was drawn, you would maximize the chance of winning the bonus by reporting a guess of 83%.

The goal of Part 2 is to familiarize with the interface where the robots communicate the correct chances. Understanding the interface is important for Part 3, as we will explain later.

Before you report your guesses, please answer the following comprehension questions.

1) Is the following statement true or false? "The robots never lie to you and never make mistakes when calculating the correct chance that bag A was selected."

a) True

b) False
2) Consider the following example:

Bag A has 1 purple balls 5 orange balls • • • • • • • • • • • • • • • • • • •							
Remember, the computer has drawn a purple ball							
The chance that bag A was selected given that the color of the drawn ball is purple is							
17%							
a) What is the correct chance (in %) that bag A was selected given that the color of the							

a) What is the correct chance (in %) that bag A was selected given that the color of the drawn ball is purple?

b) What guess (in %) would maximize your chance of winning the bonus?

## PART 2: Task 1/2

Please click on the right arrow if you are ready to proceed to the next task.

 $\rightarrow$ 

For this task, consider robot L7.



When you click "START", the following steps take place:

- Step 1: the computer fills the bags with purple and orange balls according to robot L7
- Step 2: the computer selects one bag by flipping a fair coin
- Step 3: the computer draws a ball from the bag that was selected in Step 2

START

Step 1: The computer filled the bags with balls according to robot L7.



Step 2: The computer flipped the coin and has selected the bag.



Step 3: The computer **has drawn a random ball** from the selected bag in Step 2. **The drawn ball is purple.** 



Remember that for this task we consider robot L7.



### Remember, the computer has drawn a purple ball

The chance that bag A was selected given that the color of the drawn ball is purple is...

83% What do you think is the chance that bag A was selected given that the color of the drawn

ball is purple? Make your guess using the slider

Chance of bag A (in %): Please click on the slider

## G.2.3 Part 2: Click

#### **INSTRUCTION – PART 2**

Part 2 is designed to help you understand an **important aspect of the guessing task that will be important for Part 3**: There is a **mathematical formula** (called the Bayes rule) that gives the **correct chance** that bag A was selected by the computer if you observe a ball of a certain color.

For example, consider again robot L7 that chose the color of the balls in the bags as shown in the following figure:



According to this formula:

- If the observed ball is orange, the correct chance that bag A was selected is 17%,
- If the observed ball is purple, the correct chance that bag A was selected is 83%.

In Part 2, you will complete the same guessing tasks as in Part 1, but **now the robots can inform you of the correct chance that bag A was selected given the color of the drawn ball**. Note that the robots always communicate the correct chance; that is, **the robots never lie to you and never make mistakes when calculating the correct chance**.

If you want to see this information about the correct chance, you must press the 'G' key five times in a row. Importantly, you must press the 'G' key five times consecutively if you press a different key in between, you will need to start over.



The following figure provides an example of the interface:



Again you make your guess by selecting the chance between 0% and 100% on a slider. Once the 'G' key is pressed five times in a row, a colorful pin helps you to visually indicate on the slider the correct chance communicated by the robot (see example above).

Remember that the closer your guess is to the correct chance, the higher the chance of winning the bonus of \$1. That is, **you maximize the chance of winning the bonus by submitting the correct guess provided by the robot**. In the example above, if a purple ball was drawn, you would maximize the chance of winning the bonus by reporting a guess of 83%.

The goal of Part 2 is to familiarize with the interface where the robots communicate the correct chances if you press the 'G' key five times. Understanding the interface is important for Part 3, as we will explain later.

Before you report your guesses, please answer the following comprehension questions.

1) Is the following statement true or false? "The robots never lie to you and never make mistakes when calculating the correct chance that bag A was selected."

a) True

b) False

#### 2) Consider the following example:

Bag A has 1 purple ball and 5 orange balls • • • • • • • • • • • • • • • • • • •							
Remember, the computer has drawn a purple ball							
The chance that bag A was selected given that the color of the drawn ball is purple is							
Press 'G' five times to see the chance							

You must demonstrate your understanding of how to reveal the correct chance that bag A was selected given the color of the drawn ball. To do this, press the 'G' key five times in a row to display the information in the interface. Based on the revealed information, answer the following questions.

a) What is the correct chance (in %) that bag A was selected given that the color of the drawn ball is purple?

b) What guess (in %) would maximize your chance of winning the bonus?

 $\rightarrow$ 

For this task, consider robot L7.



When you click "START", the following steps take place:

- Step 1: the computer fills the bags with purple and orange balls according to robot L7
- Step 2: the computer selects one bag by flipping a fair coin
- Step 3: the computer draws a ball from the bag that was selected in Step 2

START

Step 1: The computer filled the bags with balls according to robot L7.



Step 2: The computer flipped the coin and has selected the bag.



Step 3: The computer **has drawn a random ball** from the selected bag in Step 2. **The drawn ball is purple.** 



Remember that for this task we consider robot L7.



### Remember, the computer has drawn a purple ball

The chance that bag A was selected given that the color of the drawn ball is purple is...

Press 'G' five times to see the chance

What do you think is the chance that bag A was selected given that the color of the drawn ball is purple? Make your guess using the slider

Chance of bag A (in %): Please click on the slider

# G.2.4 Part 3: Baseline

#### **INSTRUCTION - PART 3**

You will now complete a **new version of the guessing task**. Some elements are the same as the previous version of the guessing task: there are again 2 bags, bag A and bag B; each bag contains 6 balls, and each ball is either orange or purple. Now, in each task, there are 2 robots. Each robot suggests a distribution of purple and orange balls in the bags. In the following, we illustrate the new guessing tasks with the robots H8 and Z5. Note, however, that we will use two different robots in each task. The following figure shows you how the robots H8 and Z5 would distribute the balls in the bags for this task:



**The suggestion of one of the two robots is used to fill the bags with the balls.** First, the computer randomly selects one of the two robots, either robot H8 or robot Z5, and then fills bag A and bag B according to how the selected robot would distribute the balls in the bags. The suggestion of the other robot does not count.

To select a robot, the computer randomly draws a card from a set of 100 cards and each card has an equal probability of being drawn. Each card is either red or blue. Note that the left robot is always red, and the right robot is always blue. If a red card is drawn, the red robot (H8 in the example above) is selected; if a blue card is drawn, the blue robot is selected (Z5).

Importantly, **the set of cards consists of 50 red cards and 50 blue cards**, ensuring that both robots have an equal chance of being selected. **You will not know which card is drawn and whether robot H8 or robot Z5 is selected by the computer**, and, hence, you will not know the composition of bag A and bag B.

After the computer selects a robot and fills the bags according to this robot's suggestion, the computer then flips a fair coin to select bag A or bag B as in the guessing tasks that you did in Part 1 and Part 2. It is equally likely that the computer selects bag A or bag B. As before, you will not observe which bag was selected. Instead, the computer draws one ball from the secretly selected bag and shows you the color of the ball.

As in Part 2, the robots tell you the correct chance that the observed ball was drawn from bag A, but **each robot tells you the correct chance for the case in which the robot's suggestion was selected to fill bag A and bag B**. In the example of robots H8 and Z5, consider the case in which an orange ball was drawn. In this case:

- If the robot H8's suggestion were selected, the correct chance would be 63%,
- If the robot Z5's suggestion were selected, the correct chance would be 33%.

Your goal of the new guessing task is to correctly guess the chance (in %) that bag A was selected, given the available information. Again, you make your guess by selecting the chance between 0% and 100% on a slider. Two colorful pins help you to visually indicate on the slider the correct chance if the suggestion of robot H8 is selected (63%) and the correct chance if the suggestion of robot Z5 is selected (33%):



As in the previous parts, to maximize the chance of winning the bonus of \$1, it is in your best interest to always give a guess that you think is the true chance.

In Part 3, you will complete 2 of these new guessing tasks. Note that draws in these guessing tasks are independent of any earlier draws. **So you should think about which bag was selected for each task independently of all other tasks.** 

Before you report your guesses, please answer the following comprehension questions.

1. What does the computer do in the new guessing task? (Select all options that are correct)

a) Choose the number of orange and purple balls in each bag for both robots

b) Draw a card from a set of 100 cards to select a robot

c) Flip a fair coin to select bag A or bag B

d) Randomly draw a ball from the selected bag of the selected robot

2. Is the following statement true or false? "When the computer draws a card to select a robot, you will know how many cards are red and how many cards are blue."

True			
False			

3. Consider the following example:



Remember, the computer has drawn a purple ball.

The chance that bag A was selected given that the color of the drawn ball is purple would be...



a) Suppose that the computer randomly chose robot X1 to fill bag A and bag B. What is the correct chance (in %) that the bag A was selected given that the color of the drawn ball is purple?

b) Suppose that the computer randomly chose robot H5 to fill bag A and bag B. What is the correct chance (in %) that the bag A was selected given that the color of the drawn ball is purple?

c) What do you know when you guess the chance that bag A was selected? (Select all options that are correct)

a) The number of orange and purple balls in each bag of robot X1 and of robot H5

b) Whether robot X1 or robot H5 was randomly selected to fill bag A and bag B

c) Whether bag A or bag B was selected

d) The color of the ball that is randomly drawn from the selected bag

 $\rightarrow$ 

#### PART 3: Task 1/2

Please click on the right arrow if you are ready to proceed to the next task.

 $\rightarrow$ 

For this task, consider robot P3 and robot V6.



When you click "START", the following steps take place:

- Step 1: the computer selects one of the two robots by drawing a card
  - If a red card is drawn: the computer selects robot P3
  - If a blue card is drawn: the computer selects robot V6
- Step 2: the computer fills the bags with purple and orange balls according to the robot selected in Step 1

START

- Step 3: the computer selects one bag by flipping a fair coin
- Step 4: the computer draws a ball from the bag that was selected in Step 3



Step 1: The computer drew a card and has selected the robot.

Step 1: The computer filled the bags with balls according to robot L7.



Step 2: The computer flipped the coin and has selected the bag.



Step 3: The computer **has drawn a random ball** from the selected bag in Step 2. **The drawn ball is purple.** 



Remember that for this task we consider robot P3 and robot V6.



Remember, the computer has drawn a purple ball

The chance that bag A was selected given that the color of the drawn ball is purple would be...



Chance of bag A (in %): Please click on the slider

# G.2.5 Part 3: No-Prior

#### **INSTRUCTION - PART 3**

You will now complete a **new version of the guessing task**. Some elements are the same as the previous version of the guessing task: there are again 2 bags, bag A and bag B; each bag contains 6 balls, and each ball is either orange or purple. Now, in each task, there are 2 robots. Each robot suggests a distribution of purple and orange balls in the bags. In the following, we illustrate the new guessing tasks with the robots H8 and Z5. Note, however, that we will use two different robots in each task. The following figure shows you how the robots H8 and Z5 would distribute the balls in the bags for this task:



The suggestion of one of the two robots is used to fill the bags with the balls. First, the computer randomly selects one of the two robots, either robot H8 or robot Z5, and then fills bag A and bag B according to how the selected robot would distribute the balls in the bags. The suggestion of the other robot does not count.

To select a robot, the computer randomly draws a card from a set of 100 cards and each card has an equal probability of being drawn. Each card is either red or blue. Note that the left robot is always red, and the right robot is always blue. If a red card is drawn, the red robot (H8 in the example above) is selected; if a blue card is drawn, the blue robot is selected (Z5).

Importantly, we will not disclose the composition of the card set. That is, **you will not know how many cards are red or blue. You will not know which card is drawn and whether robot H8 or robot Z5 is selected by the computer,** and, hence, you will not know the composition of bag A and bag B.

After the computer selects a robot and fills the bags according to this robot's suggestion, the computer then flips a fair coin to select bag A or bag B as in the guessing tasks that you did in Part 1 and Part 2. It is equally likely that the computer selects bag A or bag B. As before, you will not observe which bag was selected. Instead, the computer draws one ball from the secretly selected bag and shows you the color of the ball.

As in Part 2, the robots tell you the correct chance that the observed ball was drawn from bag A, but **each robot tells you the correct chance for the case in which the robot's suggestion was selected to fill bag A and bag B**. In the example of robots H8 and Z5, consider the case in which an orange ball was drawn. In this case:

- If the robot H8's suggestion were selected, the correct chance would be 63%,
- If the robot Z5's suggestion were selected, the correct chance would be 33%.

Your goal of the new guessing task is to correctly guess the chance (in %) that bag A was selected, given the available information. Again, you make your guess by selecting the chance between 0% and 100% on a slider. Two colorful pins help you to visually indicate on the slider the correct chance if the suggestion of robot H8 is selected (63%) and the correct chance if the suggestion of robot Z5 is selected (33%):



As in the previous parts, to maximize the chance of winning the bonus of \$1, it is in your best interest to always give a guess that you think is the true chance.

In Part 3, you will complete 2 of these new guessing tasks. Note that draws in these guessing tasks are independent of any earlier draws. **So you should think about which bag was selected for each task independently of all other tasks.** 

Before you report your guesses, please answer the following comprehension questions.

1. What does the computer do in the new guessing task? (Select all options that are correct)

a) Choose the number of orange and purple balls in each bag for both robots

b) Draw a card from a set of 100 cards to select a robot

c) Flip a fair coin to select bag A or bag B

d) Randomly draw a ball from the selected bag of the selected robot

2. Is the following statement true or false? "When the computer draws a card to select a robot, you will know how many cards are red and how many cards are blue."

True False

3. Consider the following example:

Bag A has 5 purple balls and 1 orange ball • • •	Bag B has 2 purple balls and 4 orange balls	Bag A has 1 purple ball and 5 orange balls • • • • • •	Bag B has 5 purple balls and 1 orange ball

Remember, the computer has drawn a purple ball.

The chance that bag A was selected given that the color of the drawn ball is purple would be...



a) Suppose that the computer randomly chose robot X1 to fill bag A and bag B. What is the correct chance (in %) that the bag A was selected given that the color of the drawn ball is purple?

b) Suppose that the computer randomly chose robot H5 to fill bag A and bag B. What is the correct chance (in %) that the bag A was selected given that the color of the drawn ball is purple?

c) What do you know when you guess the chance that bag A was selected? (Select all options that are correct)

a) The number of orange and purple balls in each bag of robot X1 and of robot H5

b) Whether robot X1 or robot H5 was randomly selected to fill bag A and bag B

c) Whether bag A or bag B was selected

d) The color of the ball that is randomly drawn from the selected bag

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### PART 3: Task 1/2

Please click on the right arrow if you are ready to proceed to the next task.

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For this task, consider robot P3 and robot V6.



When you click "START", the following steps take place:

- Step 1: the computer selects one of the two robots by drawing a card
  - If a red card is drawn: the computer selects robot P3
  - If a blue card is drawn: the computer selects robot V6
- Step 2: the computer fills the bags with purple and orange balls according to the robot selected in Step 1

START

- Step 3: the computer selects one bag by flipping a fair coin
- Step 4: the computer draws a ball from the bag that was selected in Step 3



Step 1: The computer drew a card and has selected the robot.

Step 1: The computer filled the bags with balls according to robot L7.



Step 2: The computer flipped the coin and has selected the bag.



Step 3: The computer **has drawn a random ball** from the selected bag in Step 2. **The drawn ball is purple.** 



Remember that for this task we consider robot P3 and robot V6.



Remember, the computer has drawn a purple ball

The chance that bag A was selected given that the color of the drawn ball is purple would be...



Chance of bag A (in %): Please click on the slider

## G.2.6 Part 3: Click

#### **INSTRUCTION - PART 3**

You will now complete a **new version of the guessing task**. Some elements are the same as the previous version of the guessing task: there are again 2 bags, bag A and bag B; each bag contains 6 balls, and each ball is either orange or purple. Now, in each task, there are 2 robots. Each robot suggests a distribution of purple and orange balls in the bags. In the following, we illustrate the new guessing tasks with the robots H8 and Z5. Note, however, that we will use two different robots in each task. The following figure shows you how the robots H8 and Z5 would distribute the balls in the bags for this task:



The suggestion of one of the two robots is used to fill the bags with the balls. First, the computer randomly selects one of the two robots, either robot H8 or robot Z5, and then fills bag A and bag B according to how the selected robot would distribute the balls in the bags. The suggestion of the other robot does not count.

To select a robot, the computer randomly draws a card from a set of 100 cards and each card has an equal probability of being drawn. Each card is either red or blue. Note that the left robot is always red, and the right robot is always blue. If a red card is drawn, the red robot (H8 in the example above) is selected; if a blue card is drawn, the blue robot is selected (Z5).

Importantly, **the set of cards consists of 50 red cards and 50 blue cards**, ensuring that both robots have an equal chance of being selected. **You will not know which card is drawn and whether robot H8 or robot Z5 is selected by the computer**, and, hence, you will not know the composition of bag A and bag B.

After the computer selects a robot and fills the bags according to this robot's suggestion, the computer then flips a fair coin to select bag A or bag B as in the guessing tasks that you did in Part 1 and Part 2. It is equally likely that the computer selects bag A or bag B. As before, you will not observe which bag was selected. Instead, the computer draws one ball from the secretly selected bag and shows you the color of the ball.

As in Part 2, the robots can tell you the correct chance that the observed ball was drawn from bag A, but **each robot tells you the correct chance for the case in which the robot's suggestion was selected to fill bag A and bag B**. In the example of robots H8 and Z5, consider the case in which an orange ball was drawn. In this case:

- If the robot H8's suggestion were selected, the correct chance would be 63%,
- If the robot Z5's suggestion were selected, the correct chance would be 33%.

As in Part 2, if you want to receive this information about the correct chance, you must press specific buttons on the keyboard:

- Press the 'R' key five times in a row to reveal the correct chance if the red robot were selected,
- Press the 'B' key five times in a row to reveal the correct chance if the blue robot were selected.

Importantly, you must press the same key five times consecutively—if you press a different key in between, you will need to start over. This means you cannot reveal both correct chances at the same time. You must first press the 'R' key five times before starting to press the 'B' key five times, or vice versa.

Your goal of the new guessing task is to correctly guess the chance (in %) that bag A was selected, given the available information. Again, you make your guess by selecting the chance between 0% and 100% on a slider. Once you press the 'R' and 'B' keys five times in a row, two colorful pins help you to visually indicate on the slider the correct chance if the suggestion of robot H8 is selected (63%) and the correct chance if the suggestion of robot Z5 is selected (33%):

What do you think is the chance that bag A was selected given that the color of the drawn ball is orange? Make your guess using the slider
Chance of bag A (in %): Please click on the slider

As in the previous parts, to maximize the chance of winning the bonus of \$1, **it is in your best interest to always give a guess that you think is the true chance.** 

In Part 3, you will complete 2 of these new guessing tasks. Note that draws in these guessing tasks are independent of any earlier draws. **So you should think about which bag was selected for each task independently of all other tasks.** 

Before you report your guesses, please answer the following comprehension questions.

1. What does the computer do in the new guessing task? (Select all options that are correct)

a) Choose the number of orange and purple balls in each bag for both robots

b) Draw a card from a set of 100 cards to select a robot

c) Flip a fair coin to select bag A or bag B

d) Randomly draw a ball from the selected bag of the selected robot

2. Is the following statement true or false? "When the computer draws a card to select a robot, you will know how many cards are red and how many cards are blue."

True

False

3. Consider the following example:



Remember, the computer has drawn a purple ball.

The chance that bag A was selected given that the color of the drawn ball is purple would be...

Press 'R' five times to see the chance if robot X1 were selected Press 'B' five times to see the chance if robot H5 were selected

You must demonstrate your understanding of how to reveal the left and right robots' correct chance that bag A was selected given the color of the drawn ball. To do this, press the 'R' and 'B' key five times in a row to display the information in the interface below.

Answer the following questions based on the revealed information.

a) Suppose that the computer randomly chose robot X1 to fill bag A and bag B. What is the correct chance (in %) that the bag A was selected given that the color of the drawn ball is purple?

b) Suppose that the computer randomly chose robot H5 to fill bag A and bag B. What is the correct chance (in %) that the bag A was selected given that the color of the drawn ball is purple?

c) What do you know when you guess the chance that bag A was selected? (Select all options that are correct)

a) The number of orange and purple balls in each bag of robot X1 and of robot H5

b) Whether robot X1 or robot H5 was randomly selected to fill bag A and bag B

c) Whether bag A or bag B was selected

d) The color of the ball that is randomly drawn from the selected bag

 $\rightarrow$ 

## PART 3: Task 1/2

Please click on the right arrow if you are ready to proceed to the next task.

 $\rightarrow$ 

For this task, consider robot K1 and robot X8.



When you click "START", the following steps take place:

- Step 1: the computer selects one of the two robots by drawing a card
  - If a red card is drawn: the computer selects robot K1
  - If a blue card is drawn: the computer selects robot X8
- Step 2: the computer fills the bags with purple and orange balls according to the robot selected in Step 1
- Step 3: the computer selects one bag by flipping a fair coin
- Step 4: the computer draws a ball from the bag that was selected in Step 3



Step 1: The computer drew a card and has selected the robot.

START

Step 1: The computer filled the bags with balls according to robot L7.



Step 2: The computer flipped the coin and has selected the bag.



Step 3: The computer **has drawn a random ball** from the selected bag in Step 2. **The drawn ball is purple.** 



Remember that for this task we consider robot K1 and robot X8.



Remember, the computer has drawn an orange ball

The chance that bag A was selected given that the color of the drawn ball is orange would be...

Press 'R' five times to see the chance if robot K1 were selected

Press 'B' five times to see the chance if robot X8 were selected

What do you think is the chance that bag A was selected given that the color of the drawn ball is orange? Make your guess using the slider

Chance of bag A (in %): Please click on the slider